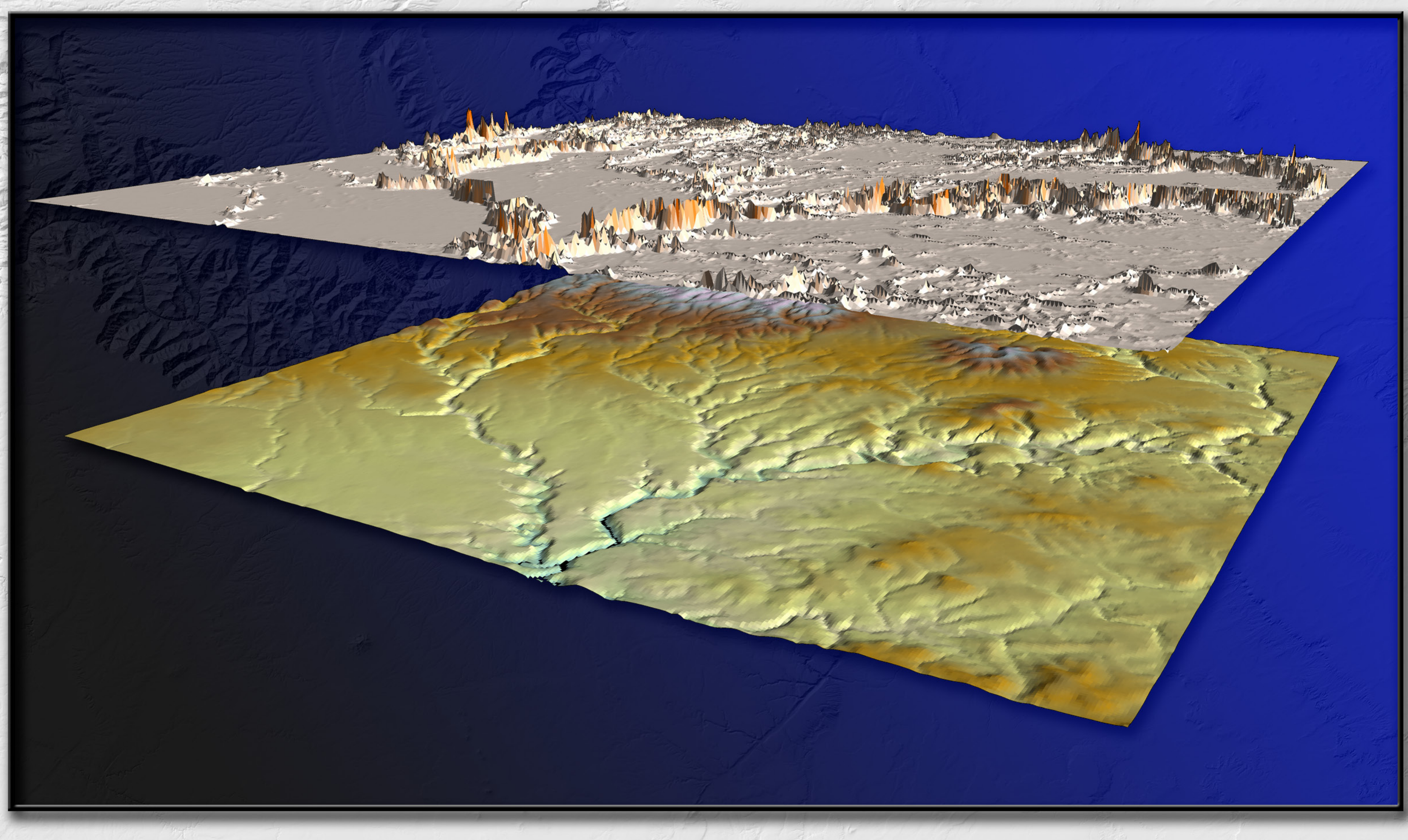
Calculating Surface Areas from Digital Elevation Models

Jeff Jenness, Wildlife Biologist
USFS Rocky Mountain Research Station
2500 S. Pine Knoll Drive
Flagstaff, Arizona 86004
jjenness@fs.fed.us



Introduction

The natural landscape, especially in the western United States, is a complex surface divided by canyons and valleys and highlighted with mountain ranges. Landscape area, however, is almost always presented in terms of planimetric area, as if a square kilometer in the Rocky Mountains of Colorado represents the same amount of land area as a square kilometer in Nebraska. Our predicted ranges for wildlife species generally use planimetric area even when we study mountain-dwelling species such as mountain goats and cougars. But if we believe that a species' behavior and population dynamics are a function of the resources available to that species, and if these resources are spatially limited, then I suggest that it might also be useful to look at the land and resources in terms of the true surface area of the landscape.

Here I demonstrate a reasonably intuitive and straightforward method for calculating surface area from Digital Elevation Models (DEMs), which are comprised of a grid of cells where each cell contains an elevation value. These DEMs are widely and freely available from various sites on the Internet

Methods

Figure 1: The surface area of each cell is calculated based on the elevation values of that cell plus the eight surrounding cells.

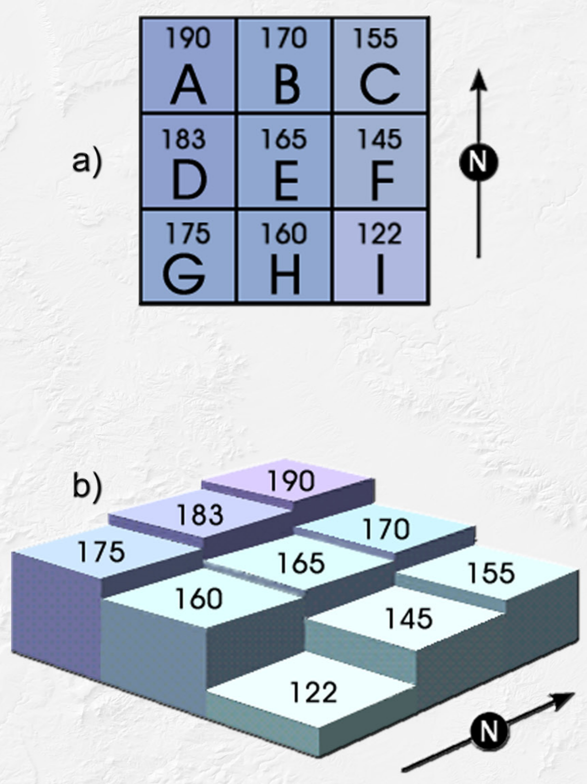


Figure 2: The cells may be pictured 3-dimensionally as a set of adjacent columns.

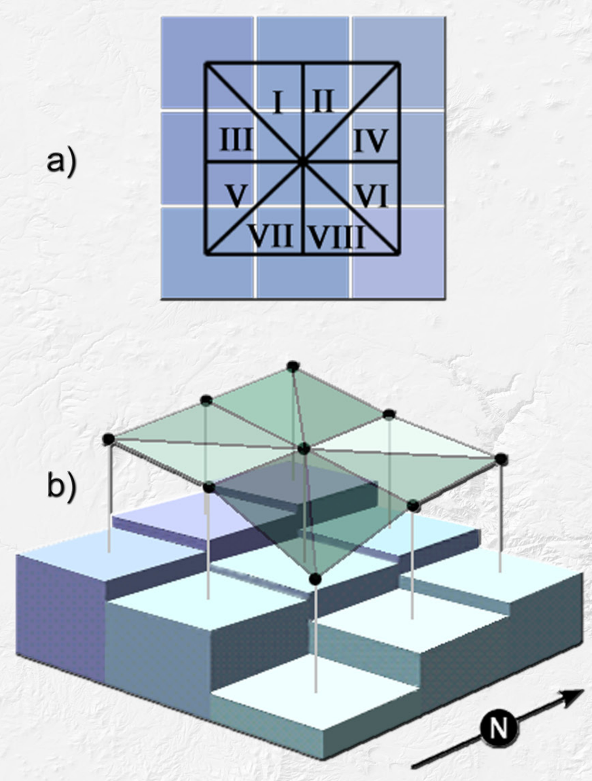


Figure 3: Surface area is calculated by adding the areas of the 8 triangles connecting the center of the central cell with the centers of the eight surrounding cells.

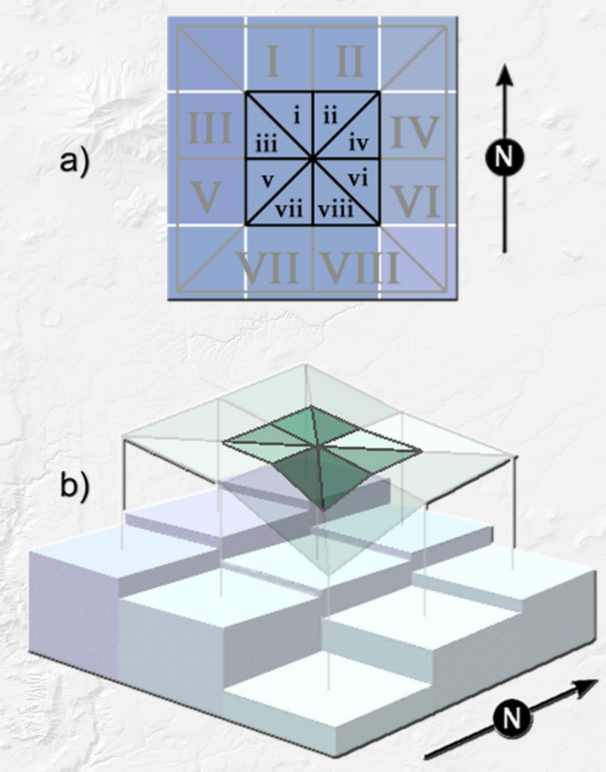
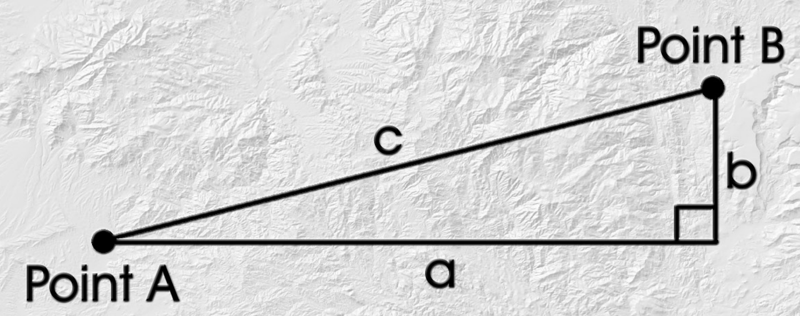


Figure 4: The triangles must be clipped to the central cell boundary so that the area only represents the region covered by that cell.

This method derives surface areas for a cell using elevation information from the eight cells adjacent to that cell. For example, given a sample elevation grid (Fig. 1), this method would calculate the surface area for the cell with elevation value "183" based on the elevation values of that cell plus the eight cells surrounding it (Fig. 1a) and the surface area for the "165" cell based on that cell plus the eight elevation value surrounding it (Fig. 1b). We can picture the cell with elevation value "165" and its surrounding elevation values in 3-dimensional space as a set of adjacent columns, each rising as high as it's specified elevation value (Fig. 2).

We take the 3-dimensional centerpoints of each of these 9 cells and calculate the surface lengths of the 8 lines that connect the central cell's centerpoint with the center points of the 8 surrounding cells. I use the term "surface length" to highlight the 3-dimensional character of this line; this is not the planimetric distance between cell centerpoints. We then calculate the surface lengths of the lines that connect each of the 8 surrounding cells with the ones adjacent to it, so that we end up with the lengths of the sides of the 8 three-dimensional triangles that all meet at the center point of the central cell (Fig. 3).

These surface lengths are simple to calculate using the Pythagorean theorem which states that, in a right triangle, the square of the length of the hypotenuse is equal to the sum of the squares of the other two sides. Thus, for any two cell centerpoints A and B:



$$a^2 + b^2 = c^2 \quad \text{or} \quad c = \sqrt{a^2 + b^2}$$

where:

a = planimetric distance from A to B
b = difference in elevation between A and B
c = surface distance from A to B

Conducting these calculations for the central cell plus the 8 adjacent cells gives us the lengths for the sides of the 8 triangles connecting the center of the central cell to the centers of the 8 adjacent cells. However, we run into a minor complication here because the triangles we have generated extend past the cell boundary and therefore represent an area larger than our cell. We must effectively trim the triangles to the cell boundaries (Fig. 4) by dividing all our length values by 2.

For example, using the elevation DEM from Figure 2, and assuming that the cells are 100 meters on a side, we begin by calculating the 16 triangle edge lengths for the 8 three-dimensional triangles radiating out from the central cell E (Fig. 3). We then divide these surface lengths in half to get the sides for triangles i-viii in Figure 4 (Table 1), and then use those lengths to determine the surface areas for each triangle (Table 2). The area of a triangle given the lengths of sides a, b and c is calculated as:

$$\sqrt{s(s-a)(s-b)(s-c)}, \qquad \text{where: } s = \frac{a+b+c}{2}$$

Table 1: The elevation values for the 9 cells in Figure 2 are used to generate 16 surface lengths for the edges of the 8 triangles in Figure 3. These surface lengths are divided in half to get the edges for the 8 triangles in Figure 4.

Triangle Edge	Planimetric Length	Elevation Difference	Surface Length	$\frac{\text{Surface Length}}{2}$
$\overline{AB}$	100	20	101.98	50.99
$\overline{BC}$	100	15	101.12	50.56
$\overline{DE}$	100	18	101.61	50.80
$\overline{EF}$	100	20	101.98	50.99
$\overline{GH}$	100	15	101.12	50.56
$\overline{HI}$	100	38	106.98	53.49
$\overline{AD}$	100	7	100.24	50.12
$\overline{BE}$	100	5	100.12	50.06
$\overline{CF}$	100	10	100.50	50.25
$\overline{DG}$	100	8	100.32	50.16
$\overline{EH}$	100	5	100.12	50.06
$\overline{FI}$	100	23	102.61	51.31
$\overline{EA}$	141.42	25	143.61	71.81
$\overline{EC}$	141.42	10	141.77	70.89
$\overline{EG}$	141.42	10	141.77	70.89
$\overline{EI}$	141.42	43	147.81	73.91

Table 2: Calculations of true surface area for triangles i-viii (Fig. 4) are based on the 16 edge lengths from Table 1.

Triangle	Edges	Edge Lengths	Triangle Area
i	$\overline{EA}$, $\overline{AB}$, $\overline{BE}$	71.81, 50.99, 50.06	1276.22
ii	$\overline{BE}$, $\overline{BC}$, $\overline{EC}$	50.06, 50.56, 70.89	1265.48
iii	$\overline{AD}$, $\overline{DE}$, $\overline{EA}$	50.12, 50.80, 71.81	1272.95
iv	$\overline{EC}$, $\overline{CF}$, $\overline{EF}$	70.89, 50.25, 50.99	1280.88
v	$\overline{DE}$, $\overline{DG}$, $\overline{EG}$	50.80, 50.16, 70.89	1273.94
vi	$\overline{EF}$, $\overline{FI}$, $\overline{EI}$	50.99, 51.31, 73.91	1306.88
vii	$\overline{EG}$, $\overline{EH}$, $\overline{GH}$	70.89, 50.06, 50.56	1265.48
viii	$\overline{EH}$, $\overline{EI}$, $\overline{HI}$	50.06, 73.91, 53.49	1338.64
		Total Surface Area for Central Cell "E"	10280.47

Additional Uses

Neighborhood analysis: In many cases we are not interested in values of individual cells but rather the values in a region around those cells. This is especially common when we are interested in phenomena over multiple spatial scales. For example, we may be interested in knowing, say, the average surface area within 1 km for all points on the landscape. Using neighborhood analyses, we could specify a neighborhood of 1 km and then calculate the average surface area value within that neighborhood for each cell. The cell values in the resulting grid, regardless of the sizes of the cells, would then reflect the average surface area within 1 km of each cell. Changing the neighborhood value to 100 km gives us an average surface area grid for a much broader spatial scale. Such neighborhoods can take on a variety of shapes, including squares, doughnuts, wedges and irregular shapes.

Surface area ratio grids: Surface Area grids may easily be standardized into Surface Area Ratio grids by dividing the surface area value for each cell by the planimetric area within that cell. These surface area ratio grids are useful as a measure of topographic roughness over an area and could conceivably be used as friction or cost grids for analysis of movement (such grids would steer the predicted direction of movement based on the topographic roughness of a cell). Because these ratio grids are in raster format, they also lend themselves to neighborhood-based statistics as described above.

More accurate proportions of available resources: By weighting our resource maps with the underlying surface area values, we generate more accurate extents and proportions of our resources within a particular region. This is especially true if any of our resources are especially associated with particularly steep or flat areas.

Testing and Validation

I tested the accuracy of this method by calculating the surface area within a large number of randomly-generated polygons scattered over central Arizona and eastern New Mexico, and comparing the surface area values with values computed using Triangulated Irregular Networks (TINs). Details of these tests are available from the author or from *Calculating landscape surface area from digital elevation models* (Wildlife Society Bulletin, In Press). In short, I found that this method produces highly accurate surface area measures provided that a sufficiently large number of elevation cells were used in the analysis (for most purposes, $n > 250$ may be sufficient), with accuracy increasing as the proportion of cells completely contained within the polygon boundary increases.

For ArcView 3.x Users

For users of ESRI's ArcView 3.x software with Spatial Analyst, the I have made available a free extension which automates this process and directly produces surface area and surface ratio grids from grid-formatted DEMs. This extension may be downloaded from the author's web site at <http://www.jennessent.com/arcview/surface_areas.htm> or from the ESRI ArcScripts site at <http://arcscripts.esri.com/details.asp?dbid=11697>.