

Issues with Directional Data

Many characteristics of wildlife habitat and behavior include a directional component. Animal movement may tend to angle toward or away some landscape feature, and the strength of the attraction or repulsion effect may change with proximity to that object. Aspect (i.e. the azimuth the landscape faces in the direction of steepest slope) is an intuitively valuable habitat variable because north-facing slopes are typically cooler and more mesic while south-facing slopes are generally warmer and more xeric (at least in the northern hemisphere!). Habitats can be dramatically different depending on which side of a hill you stand on.

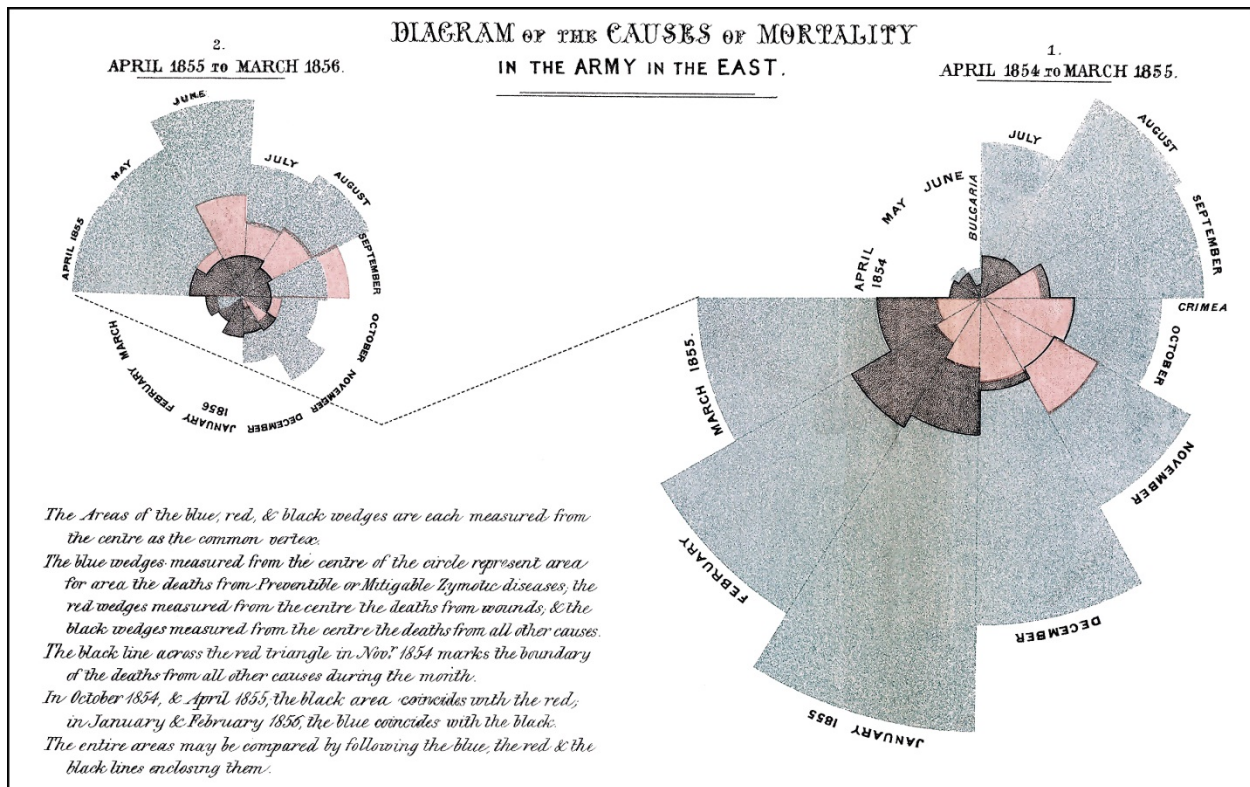
Direction can be difficult to analyze statistically because of its circular nature. The difference between 1 degree and 360 degrees is the same as the difference between 1 degree and 2 degrees, making it difficult to plug into a typical statistical model. This article will discuss some issues we often face with directional data plus some basic ideas to describe and analyze it. Many of the concepts can also be applied to other periodic data such as time, but this article will focus on direction.

An additional complication is that sometimes direction should be counted as a single value (the “forward” direction) and sometimes both the forward and reverse directions should be counted. In wildlife analysis, direction is usually treated as a single value. We are interested in where an animal is going and why it chooses to go in the direction it does. Or we are interested in the general aspect of a habitat block, and what that means to our species of interest. In some cases, however, we need to consider both the direction and its opposite value. Analysis of geologic fracture lines on the landscape should be considered to go in both directions. Analysis of trails or paths should be treated similarly, except in the rare case of one-way trails.

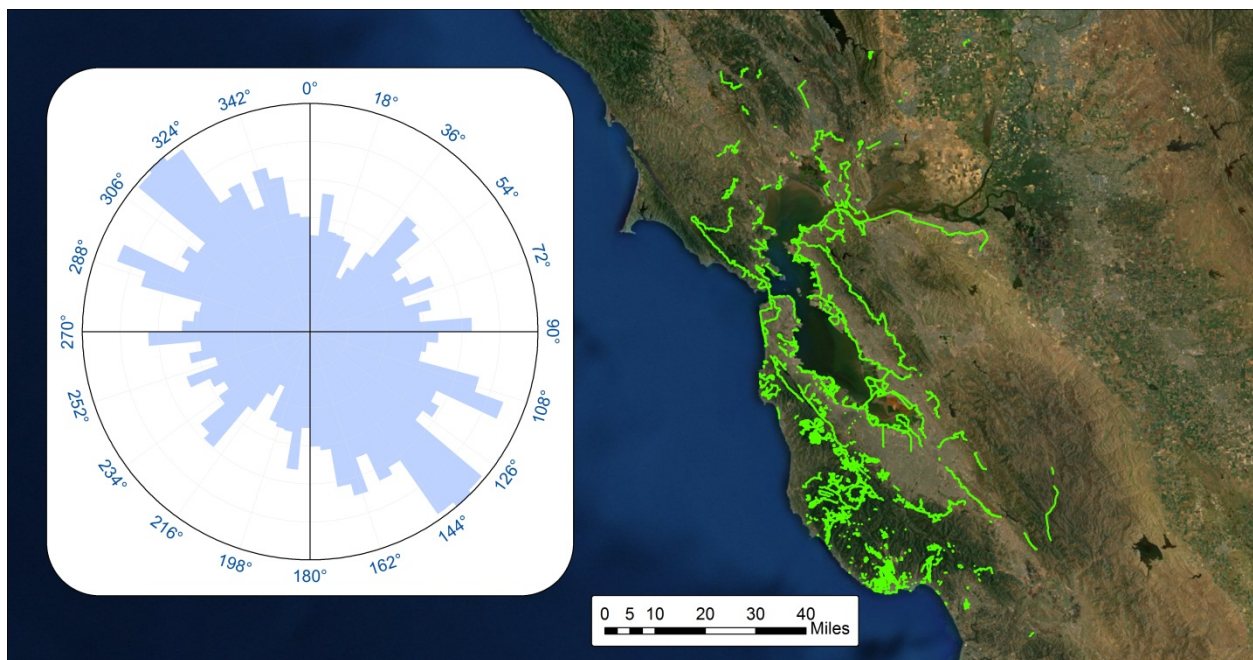
Graphically Representing Directional Data

Because of the circular nature of directional data, we typically use circular plots to display the distribution. Most of these circular plots are variations on the standard histogram, except wrapped in a circle so that the maximum possible X-axis value wraps back to touch the minimum possible value.

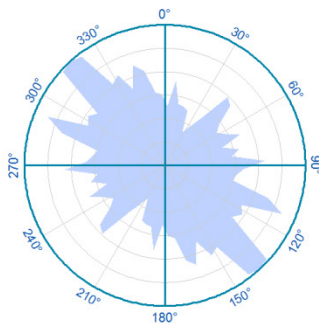
One of the more famous and historically interesting of these is the Rose Plot or Rose Diagram, named as such because the histogram bins resemble flower petals. This type of diagram was originally popularized by nurse and statistician Florence Nightingale, who used it to show seasonal patterns in soldier mortality (see http://en.wikipedia.org/wiki/Florence_Nightingale for the source of the famous image below). Nightingale actually called these diagrams “coxcombs” because they resemble the comb of a rooster.



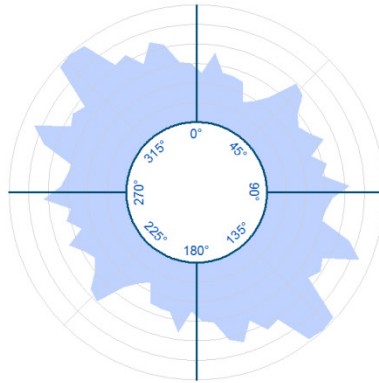
Nightingale's plots showed variation by season, but this method also works very well for displaying directional data. For example, given a set of trails in the San Francisco Bay area, we can use a rose plot to illustrate the general Northwest/Southeast orientation of those trails:



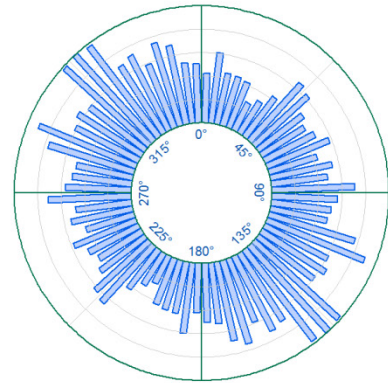
Common variations on the traditional rose plot include symbolizing the histogram bins as "Peaks and Valleys", and putting the bins on the outside of the circle rather than the inside.



Peaks and Valleys

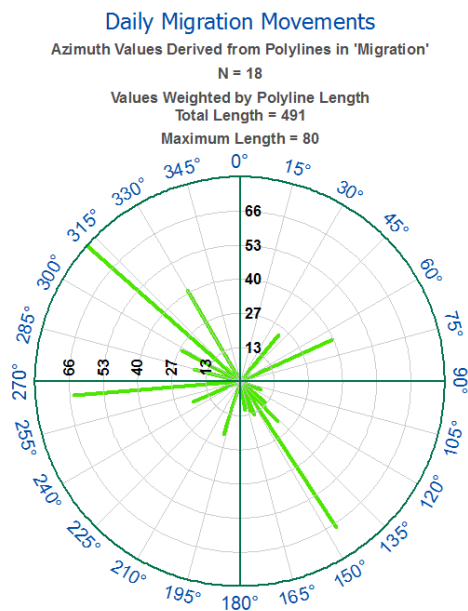


External Peaks and Valleys

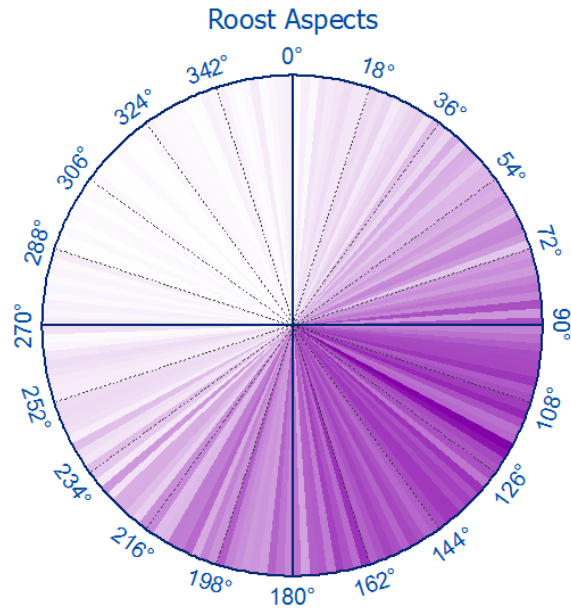


External Histogram Bins

When only a few direction values need to be displayed, you can simply show the bearings in a circular plot.



Another interesting method is a pie chart-type image where the “slices” are shaded according to the proportion of the data that faces that particular direction:



Although this last method is less effective at showing the shape of the distribution as other methods, it is still an interesting style of illustration because it is so visually intuitive. In the image above, it is easy to see that the observed roost locations predominately face toward the southeast.

For those who are interested, the author has provided a free ArcGIS extension to create all the plot types illustrated above (see http://www.jennessent.com/arcgis/polar_plots.htm). Additional software tools can be found with a Google search. Fisher (1995:15-30) also discusses alternative methods for graphing circular data.

Analyzing Directional Data

Circular Statistics

Before beginning this section, I want to emphasize two important points that people very often make mistakes with:

- 1) **Do not calculate the mean direction using the arithmetic mean!** This is especially frustrating because the arithmetic mean is sometimes correct and sometimes wildly incorrect. For example, the mean direction of 90° and 180° is 135°, which coincidentally is equal to the arithmetic mean $\left(\frac{90+180}{2} = 135\right)$. But what is the mean direction of 359° and 1°? They are both pointing almost due north ($\pm 1^\circ$), and clearly the true mean direction is exactly due north. However, the arithmetic mean gives us 180° $\left(\frac{359+1}{2} = 180\right)$, which is due *south* and exactly the opposite of the correct answer. The correct way to calculate the mean direction is described below.
- 2) **If you apply a Sine or Cosine transformation, make sure to convert the values to radians first!** Most analysis software and programming languages have sine and cosine functions, but these functions usually assume the values are in *radians*, not *degrees*. There are exactly 2π (~6.28)

radians in a circle. This means that the software will assume that a difference of 3.14 (i.e. π) units is equivalent to going halfway around the circle. If your data are in degrees, then the software will interpret a change in 2° is roughly equivalent to going a third of the way around the circle. Fortunately it is easy to convert to radians using the following formula:

$$\text{Degrees to Radians: } \text{Radians} = \frac{\text{Degrees} \times \pi}{180} = \frac{\text{Degrees} \times (3.14159265358979)}{180}$$

It is possible that your software has a function that allows you to calculate sines and cosines from degrees (many calculators do), but even in this case you must remember to set the switch correctly.

Fortunately there are well-established methods available for analyzing circular or periodic data such as movement direction or aspect. The circular nature of the data lead to very specific and interesting approaches to calculating measures of central tendency and dispersion. Some of the basic descriptive statistics are:

Mean Direction and Mean Resultant Length

Where: θ = Direction in Radians

$$C = \sum_{i=1}^n \cos \theta_i \quad S = \sum_{i=1}^n \sin \theta_i \quad R^2 = C^2 + S^2$$

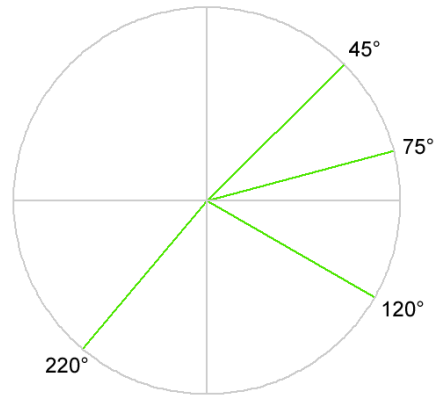
$$\text{Mean Direction: } \bar{\theta} = \begin{cases} \tan^{-1}(S/C) & S > 0, C > 0 \\ \tan^{-1}(S/C) + \pi & C < 0 \\ \tan^{-1}(S/C) + 2\pi & S < 0, C > 0 \\ \pi/2 & S > 0, C = 0 \\ -\pi/2 & S < 0, C = 0 \end{cases}$$

$$\text{Resultant Length: } R = \sqrt{R^2}$$

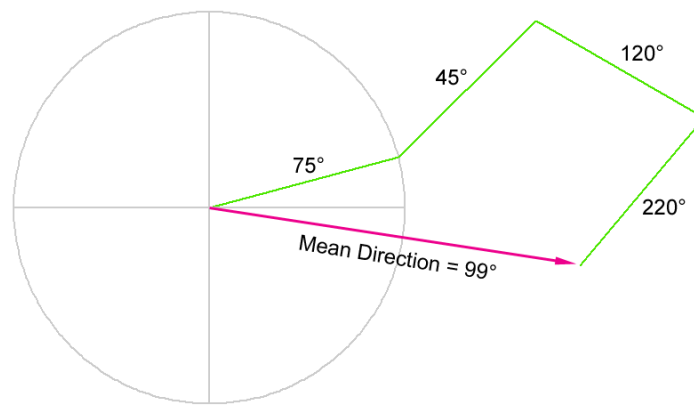
$$\text{Mean Resultant Length: } \bar{R} = \frac{R}{n}$$

The equations for mean direction look a little confusing, but the logic is actually very intuitive. It is simply a process of vector addition, where each direction value is a single vector. Vector addition essentially connects all the direction vectors into a path, and the mean direction is just the direction to the last point on the path.

For example, consider a scenario with 4 direction values at 45° , 75° , 120° and 220° :

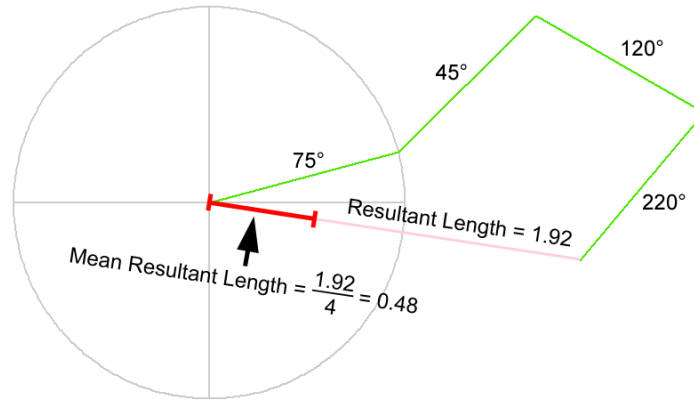


We connect the 4 bearings in a path (vector addition just adds up the ΔX and ΔY components of each vector, which is the same as treating each bearing as a segment in a path). It does not matter what order we connect the vectors in; they will always end up at the same point. The **Mean Direction** is the bearing from the start of the path to the end of the path.



On a side note, this is also the way to calculate the mean direction of an actual observed movement path. If you have a series of locations from a GPS collar on an elk, for example, and you wonder what average direction the animal moved over the day, then that average direction is simply the direction from the first GPS location of the day to the last.

The **Mean Resultant Length** $\bar{R}$ is the basis for several values of dispersion (analogous to variance or standard deviation), and is calculated as the straight-line distance from the starting point to the ending point of the path divided by the number of segments (if the segments are unit vectors, where each vector has a length of 1) or by the total length of the path if the segments are not unit vectors.



Notice that the mean resultant length has a range of 0 to 1. If all the vectors pointed in exactly the same direction, the *resultant length* would then be equal to the total path length and the *mean resultant length* would be equal to 1. This is the scenario with the minimum possible variance or dispersion in the vectors. The more the path wanders around, the shorter both the *resultant length* and the *mean resultant length* will be. If the path ended back at the origin, then both values would be equal to 0.

Variance and Standard Deviation:

Mardia and Jupp (2000), Fisher (1995) and Batschelet (1981) describe circular statistics analogous to variance and standard deviations, based on the mean resultant length.

$$\text{Circular Variance: } V = 1 - \bar{R} \quad (\text{Mardia \& Jupp, Fisher})$$

$$\text{Angular Variance: } S^2 = 2(1 - \bar{R}) \quad (\text{Batschelet})$$

$$\text{Circular Standard Deviation: } v = \sqrt{-2 \ln \bar{R}} \quad (\text{In Radians}) \quad (\text{Mardia \& Jupp, Fisher})$$

$$\text{Angular Deviation: } s = \sqrt{S^2} = \sqrt{2(1 - \bar{R})} \quad (\text{In Radians})(\text{Batschelet})$$

The *Circular Standard Deviation* and *Angular Deviation* are both in units of Radians, but these can easily be converted to Degrees.

$$\text{Radians to Degrees: } \text{Degrees} = \frac{180}{\pi} (\text{Radians})$$

Note: A mean resultant length near 1 always implies a tightly focused set of directions, but a mean resultant length near 0 does **not** necessarily imply a high amount of variation or dispersion. All it implies is that the directions balance each other out. This can occur with a uniform distribution of directions, in which case there truly would be high dispersion. It can also occur, however, if you have a number of bearings in one direction and an equal number in the opposite direction. For example, perhaps you have a bird with distinct and separate roost and forage locations. Every day the bird travels to the forage location and then returns to the roost location. In this scenario, the bird only goes in two distinct and opposite directions (to the forage location, and then back to the roost location). The bird's movement directions are highly focused and predictable, but the mean resultant length would be 0 and consequently the variance would be high.

Just as with standard statistics, there are a number of circular distributions and sophisticated analytical techniques available. These go beyond the scope of this newsletter article, but please refer to Jammalamadaka and SenGupta (2001), Mardia and Jupp (2000), Zar (1999; see especially ch. 26 and 27), Fisher (1995) and Batschelet (1981) for some good texts on circular statistics, distributions (i.e. the

Fisher, Von Mises and Wrapped Normal distributions), circular hypothesis testing and other analytical techniques. There is also a good circular statistical package for R, originally based on Jammalamadaka and SenGupta's text. As of June, 2012, the manual for this package can be viewed at <http://cran.wustl.edu/web/packages/circular/circular.pdf>.

Other Methods to Analyze Directional Data

If we want to include direction as one of several independent predictor variables in a traditional statistical analysis, then we must transform the value into something approaching a linear measure. There are a few straightforward ways to do this, although any transformation will probably violate some of the assumptions of most traditional statistical tests.

Classification of Aspect Values

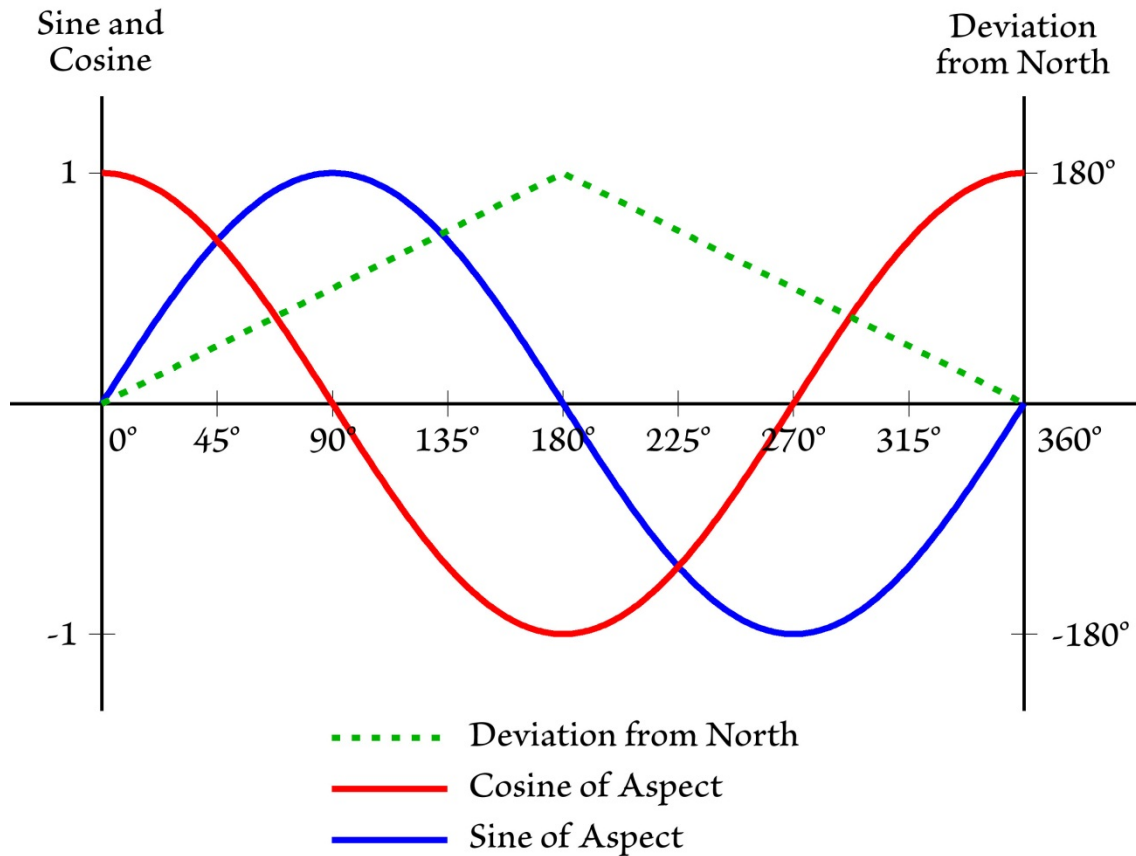
Probably the easiest transformation is to simply group your direction values into general and intuitive ranges (for example, "N" = 315 – 45, "E" = 45 – 135, "S" = 135 – 225, and "W" = 225 - 315), creating a categorical dataset which may be appropriate for some analyses.

Deviations from a Bearing

A simple and basic transformation is to convert your direction values into deviations from a direction of interest. For example, if you felt that the object of your study was likely to be affected by the north- vs. south-facing aspect phenomenon, then you might define your direction values in terms of "Deviation from North" where each value would reflect the distance, in degrees, from due North. Your full set of transformed values would range from 0 to 180 (see figure below). This option has the advantage of maintaining a constant interval between units, such that the difference in direction between 0 and 1 degree is the same as the difference between 90 and 91 degrees.

Sine and Cosine Transformations

Aspect values are often converted to sine and cosine values, essentially decomposing them into north-south and east-west components. *Remember to convert your direction values to radians before doing this transformation!* Sine values range from -1 (at due west) to 1 (at due east), while cosine values range from -1 (at due south) to 1 (at due north). Note that this method does not maintain a constant interval between units. The sine and cosine values change by a variable amount depending on the direction, such that a change in sine corresponding to a change of 1 degree = 0.00015 when going from 90 to 91 degrees, but increases by more than 2 orders of magnitude to 0.017 when going from 180 to 181 degrees. This issue may be important in your statistical analysis if your method assumes that your data are interval-level.



Trimble and Weitzman (1956) and Beers et al. (1966) suggest an interesting alternative combining two of the approaches above, rescaling aspect values based on an optimum bearing (135° for Trimble & Weitzman, and a general equation for any aspect of interest for Beers et al.) then taking the sine of the rescaled values, and adding 1, which they put to extensive use in site productivity research for timber stands. Beers et al.'s general equation is:

$$\begin{aligned}
 A' &= \text{Transformed Aspect Code} \\
 &= \sin[A + (90 - A_{Max})] + 1 \\
 &= \cos(A_{Max} - A) + 1
 \end{aligned}$$

Where:

A_{Max} = The aspect which is to be assigned the highest numerical value on the transform scale

Special Considerations for Aspect

How Aspect is Calculated

Aspect is calculated from the directional East-West and North-South gradients at a particular point. There are various methods for estimating these directional gradients from a 3x3 set of elevation points, but in general aspect is defined as the direction of maximum slope. Given that G = east-to-west gradient and H = north-to-south gradient, then aspect is calculated as follows:

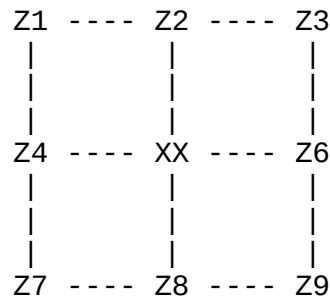
$$\text{Mathematical Direction (in degrees)} = \text{atan2}(H, -G) \left(\frac{180}{\pi} \right)$$

Mathematical direction is different than compass direction. In the mathematical polar coordinate system, 0 degrees is equivalent to due east and polar directions increase in a counter-clockwise direction. Compass direction starts at north and increases in a clockwise direction. Therefore we must convert the mathematical direction to a compass aspect according to the following rules:

if *Mathematical Direction* > 90° then *Aspect* = (450 – Direction)

Otherwise *Aspect* = (90 – Direction)

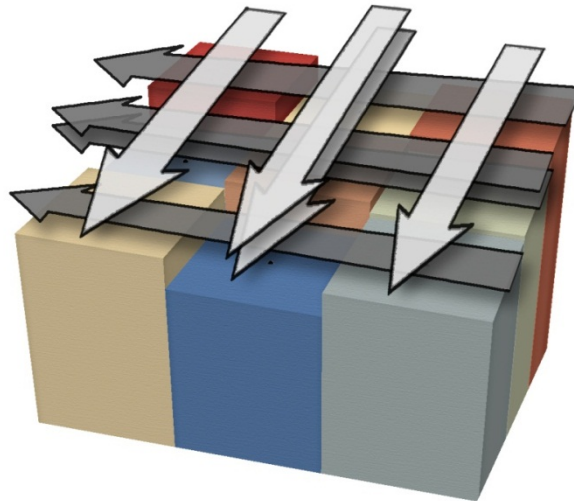
ArcGIS uses Horn's method (Horn 1981, Burrough and McDonnell 1998) to calculate the east-west (G) and north-south (H) gradients from a 3x3 array of elevation points. The figure below shows the 8 elevation values that are used to calculate aspect at the central cell XX (note that the formula does not use the elevation value at the central cell):



In Horn's Method, the north-south gradient is calculated from all three columns in the 3x3 array, with the central column weighted twice as high as the two outer columns. The east-west gradient is calculated in a similar manner.

$$\text{East-West Gradient} = G = \frac{[(Z_3 + 2Z_6 + Z_9) - (Z_1 + 2Z_4 + Z_7)]}{8\Delta x}$$

$$\text{North-South Gradient} = H = \frac{[(Z_1 + 2Z_2 + Z_3) - (Z_7 + 2Z_8 + Z_9)]}{8\Delta y}$$



Is this the best method to calculate aspect?

Interestingly, Jones (1998) compared several slope and aspect algorithms on a mathematical surface in which the true exact slope and aspect at any point could be calculated. He found that Horn's method did better than most methods, but was marginally worse than a method called the 4-cell method. He then tested all methods on a true landscape DEM and found that the rankings among methods were consistent, so Horn's method still came in 2nd place.

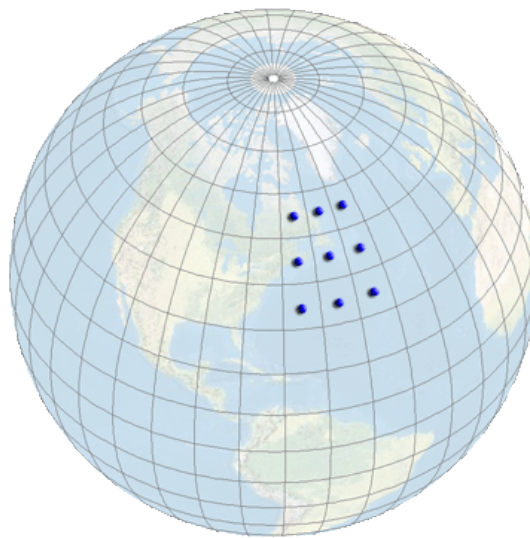
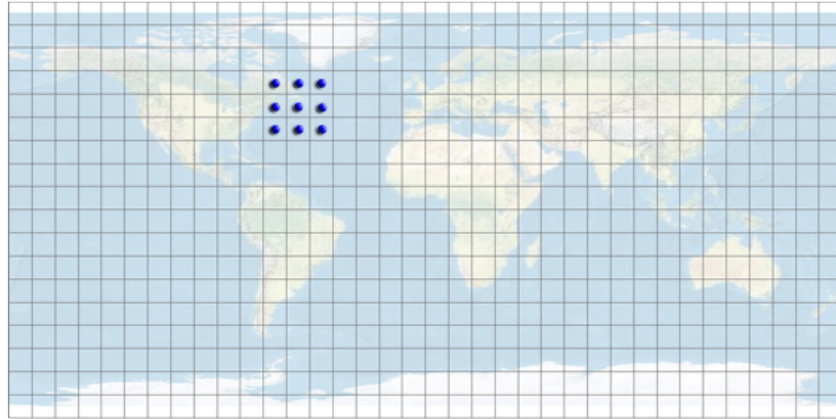
Another tidbit: Michael Hodgson (1995) published an interesting paper using the same mathematical surface that Jones (1998) did, in which he demonstrates that the area represented by computed slope and aspect is actually larger than the raster DEM cell size. 8-cell algorithms (such as *Horn's* method) more accurately reflect values for an area approximately twice the cell size, while 4-cell algorithms (such as the *4-Cell* method) reflect an area approximately 1.6X the original cell area.

In sum, a method called the 4-cell method is slightly quicker to calculate, more precise and marginally more accurate than Horn's method.

For those who are interested, the author offers a free ArcGIS extension that will calculate slope and aspect using a variety of methods (see http://www.jennessent.com/arcgis/surface_area.htm). This extension does not require Spatial Analyst.

Issues with Aspect

As a consequence of determining aspect from East-West and North-South components, aspect can be difficult to calculate from geographically-projected (latitude/longitude) data. The equations above all work for 3x3 arrays in which the elevation locations are all equally spaced along both the X- and Y-axes. However, this is not the case with latitude / longitude data. Raster cells that are square in unprojected space actually form trapezoids when viewed on the actual surface of the planet.



In this case, we cannot treat the cell size as a single constant value. The methods to correct for latitude/longitude data go beyond the scope of this article, but are described in Jenness (2011; see especially p. 40 - 49 of the manual at <http://www.jennessent.com/downloads/DEM%20Surface%20Tools%20for%20ArcGIS.pdf>). For those who are interested, the author's free ArcGIS extension DEM Surface Tools will automatically correct for latitude/longitude data (see http://www.jennessent.com/arcgis/surface_area.htm).

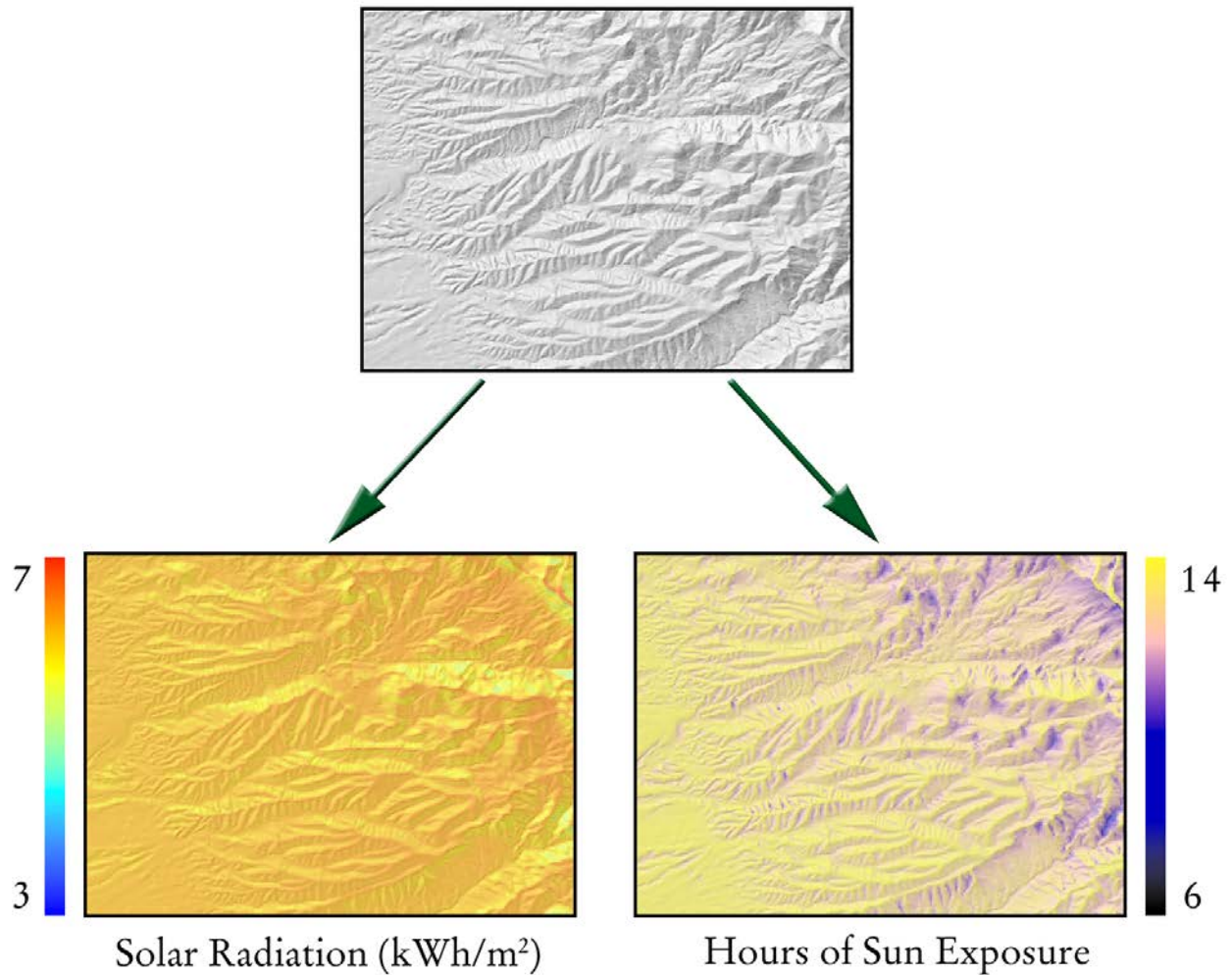
Solar Insolation: An alternative to Aspect

Aspect has a well-established history in habitat analysis. It is both easy to measure and a good predictor of certain habitat characteristics. However, in some cases we are really interested in how much direct sunlight hits an area (insolation), which is a function of aspect, slope, nearby topography, landscape reflectivity and atmospheric effects, and as such may be a more important driver of habitat characteristics than aspect alone. In such cases, it may be worthwhile to estimate insolation directly rather than use aspect as a surrogate. There are a few approaches you can take:

ESRI's Solar Radiation Tool

As of version 9.2 of ArcGIS and Spatial Analyst, ESRI has included a tool in the ArcToolbox to calculate solar radiation for specific points on the landscape, or over the entire landscape.

Digital Elevation Model



This function incorporates both direct and diffuse radiation and shadows from nearby topography. It includes several useful options, including:

- 1) Outputs either the amount of energy hitting the ground, in Watts per Hour, or the total number of hours in which the ground is exposed to the sun.
- 2) Can be calculated for specific dates, seasons or years.
- 3) Has optional parameters where you can specify the general atmospheric conditions in your area of interest.
- 4) Has optional parameters where you can specify how intensively it examines the local topography before determining the amount of radiation hitting an area.

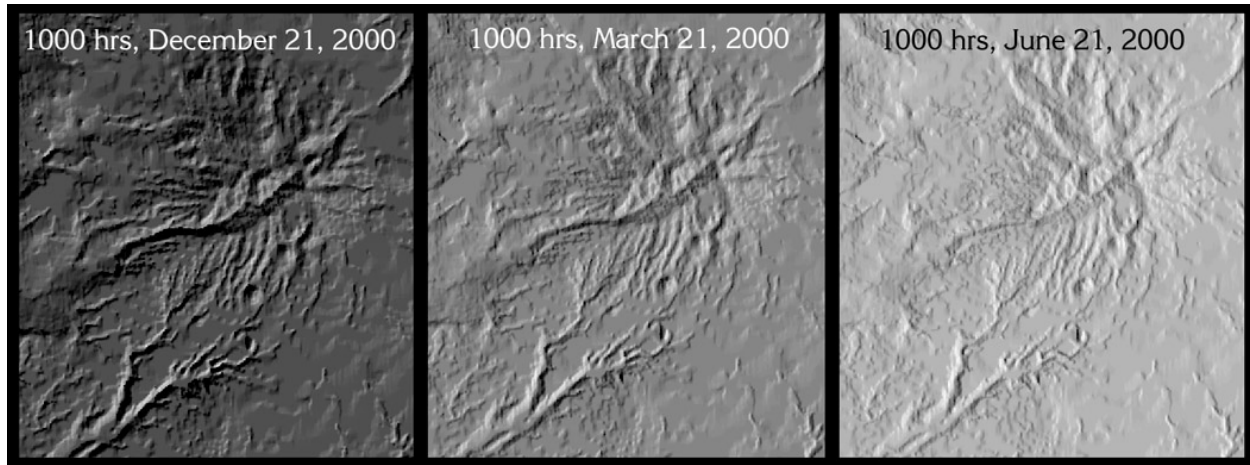
In general, this is a wonderful and exciting new tool. I have noticed two minor drawbacks to the tool:

- 1) It is slow on large grids, and on occasion I have needed to let it run for hours or days.
- 2) I do not believe that it incorporates reflectivity off the landscape. This would be hard to model, of course, and would depend on exactly how reflective your landscape is (snow reflects very differently from lava rocks, for example).

Despite these two minor drawbacks, I expect that this tool will become very valuable for habitat analysis.

Hillshade

For those who do not have access to the Solar Analyst, you can use the hillshade function available in many GIS packages get a reasonable estimate for the relative amount of sunlight that hits the landscape at a single point in time. Values will typically range from 0 (meaning no sunlight hits the landscape) to 255 (meaning the angle of the landscape is facing the sun directly). Hillshades are difficult to calculate if your DEM is in geographic coordinates, but the author offers a free ArcGIS tool to do this (see http://www.jennessent.com/arcgis/surface_area.htm) which does not require Spatial Analyst.



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