

**NAME:** Center of Mass, v. 1.b

**Aka:** center\_of\_mass.avx

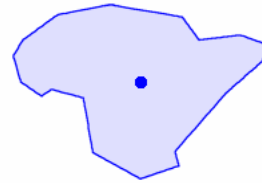
**Last modified:** September 28, 2006

**TOPICS:** Polygon, Centroid, Center of Mass, Vector, Area

**AUTHOR:** Jeff Jenness

Wildlife Biologist, GIS Analyst  
Jenness Enterprises  
3020 N. Schevene Blvd.  
Flagstaff, AZ 86004 USA

Email: [jeffj@jennessent.com](mailto:jeffj@jennessent.com)  
Web Site: <http://www.jennessent.com>  
Phone: 1-928-607-4638



**DESCRIPTION:** Polygon centroids produced within ArcView (using the "ReturnCenter" request) do not always represent the true center of mass of that polygon. This extension provides a simple way to calculate both the true centroid of a polygon and the ESRI version of the centroid.

**All or only selected records:** You can either use all the polygons in the theme for the analysis or only a selected subset of polygons. If any polygons in your theme are selected, then only those selected polygons will be used to generate the centroids. If no polygons in your theme are selected, then all polygons will be used in the analysis.

**Projected vs. Unprojected Views:** If your original polygons are in Lat/Long coordinates (the Geographic Projection) and your View has been projected, then you have the option of calculating centroids based on either the Geographic projection or your View projection. The choice of projections can dramatically affect the location of the centroids.

**Output:** This extension will generate a point theme representing the polygon centroids. The point theme will contain all the original attribute fields of the original polygon theme.

**Requires:** This extension requires that the file "avdlog.dll" be present in the ArcView/BIN32 directory (or \$AVBIN/avdlog.dll) and that the Dialog Designer extension be located in your ArcView/ext32 directory, which they usually are if you're running AV 3.1 or better. The Dialog Designer doesn't have to be loaded; it just has to be available. If you are running AV 3.0a, you can download the appropriate files for free from ESRI at:

<http://support.esri.com/index.cfm?fa=downloads.patchesServicePacks.viewPatch&PID=25&MetaID=483>

**Revisions:** Version 1.a (December 8, 2004) makes very minor changes to the code, making it run slightly more efficiently and adding X- and Y-coordinates to the centroid attribute table. This version also corrects an error in the Manual equations.

Version 1.b (September 28, 2006) fixes an error in which the output shapefiles had two fields labeled "X\_Coord" rather than a single "X\_Coord" and a single "Y\_Coord" field.

**Recommended Citation Format:** For those who wish to cite this extension, the author recommends something similar to:

Jenness, J. 2006. Center of Mass (center\_of\_mass.avx) extension for ArcView 3.x. Jenness Enterprises. Available at: <http://www.jennessent.com/arview/centermass.htm>.

Please let me know if you cite this extension in a publication ([jeffj@jennessent.com](mailto:jeffj@jennessent.com)). I will update the citation list to include any publications that I am told about.

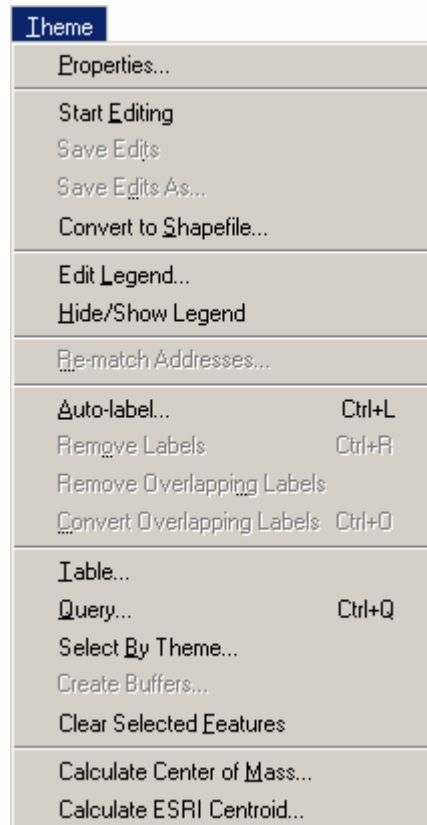
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## Table of Contents:

|  |    |
|--|----|
| General Instructions: .....  | 2  |
| General Concepts: .....  | 3  |
| How to Calculate the True Centroid (for those who want all the details): ..... | 5  |
| Calculating Triangle Areas and Centroids: .....                                | 9  |
| Notes for Avenue Programmers: .....  | 11 |
| References: .....  | 13 |

## General Instructions:

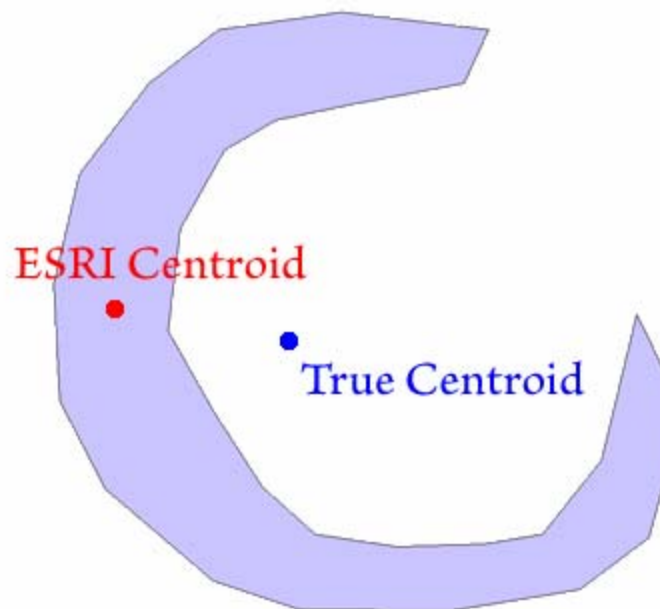
- 1) Begin by placing the "center\_of\_mass.avx" file into the ArcView extensions directory (../Av\_gis30/Arcview/ext32/).
- 2) After starting ArcView, load the extension by clicking on File --> Extensions... , scrolling down through the list of available extensions, and then clicking on the checkbox next to the extension called "Center of Mass."
- 3) The two Centroid functions can be found at the bottom of the *Theme* menu in the View menu bar. These menu items will only be enabled if you have a single polygon theme active in your view.



## General Concepts:

The centroid of a polygon is generally defined as the center of mass of that polygon, such that if some flat substance of uniform thickness and density were cut into the shape of that polygon, then that object would balance at the centroid.

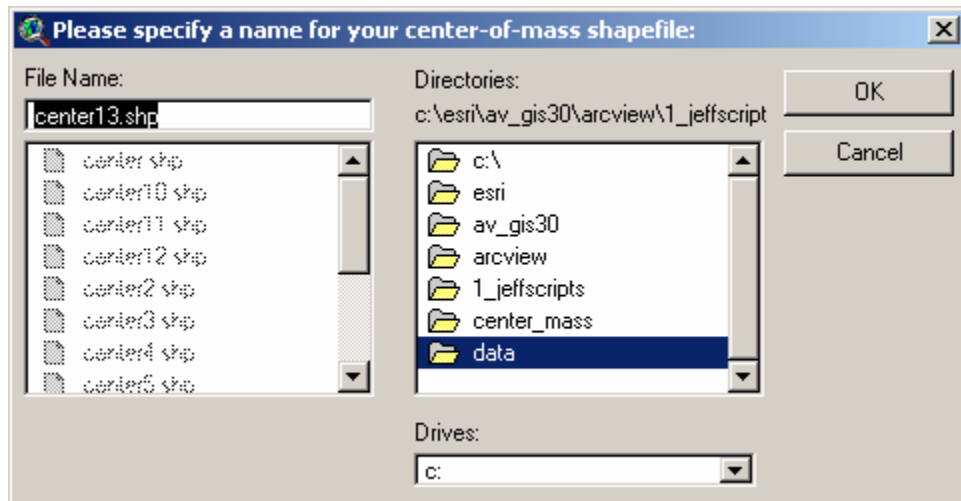
The ESRI centroid does not always fit this definition. I am unsure of exactly what logic or algorithm the ESRI code uses to generate the centroid, but I am sure it does not always produce the center of mass. An important difference between the ESRI centroid and the true centroid is that the ESRI centroid will always be contained within the polygon boundary, while the true center of mass may be outside the boundary. For example, consider a polygon shaped like a letter C. The true center of mass is actually between the arms of the C:



I suspect that ESRI originally intended to produce an anchor point to attach a polygon label to, and therefore wrote their code to make sure the anchor point was always positioned within the polygon. This would be a reasonable thing to do and this process does produce a useful point. However, by labeling this point the "centroid", they incorrectly imply that this point is the center of mass of that polygon. Some analyses may need the true centroid of the polygon while others may just need a representative point inside that polygon, and this extension provides both options.

To calculate either the true center of mass of a polygon or the ESRI centroid, click on your polygon theme in your View table of contents to make it active, and then click the appropriate menu item in the *Theme* menu bar. The menu items will only be enabled if you have a single polygon theme active in your view.

You will be prompted to name your new Centroid theme:



Click "OK" and the extension will generate a new point theme in your view. The point theme attribute table will contain all the attribute fields from your original polygon theme, plus 4 new fields containing a unique ID value, the theme name of the original polygon theme, and the X- and Y-coordinates of the points.

## How to Calculate the True Centroid (for those who want all the details):

The mathematical details can be summarized as follows:

$$\text{Polygon Centroid X-coordinate} = \frac{\sum_{i=1}^{n-1} A(\Delta V_i V_{i+1} P) g_x(\Delta V_i V_{i+1} P)}{\sum_{i=1}^{n-1} A(\Delta V_i V_{i+1} P)}$$

$$\text{Polygon Centroid Y-coordinate} = \frac{\sum_{i=1}^{n-1} A(\Delta V_i V_{i+1} P) g_y(\Delta V_i V_{i+1} P)}{\sum_{i=1}^{n-1} A(\Delta V_i V_{i+1} P)}$$

where:

$n$  = # vertices in polygon

$P$  = a new point at coordinates  $P_{x,y}$ , location is not important.

$A(\Delta V_i V_{i+1} P)$  = Signed Area (+ or -) of triangle defined by vertices  $i$ ,  $i+1$ , and  $P$

$$= \frac{(V_{(i+1)x} - V_{(i)x})(P_y - V_{(i)y}) - (P_x - V_{(i)x})(V_{(i+1)y} - V_{(i)y})}{2}$$

$g_{x,y}(\Delta V_i V_{i+1} P)$  = X- or Y-coordinate of centroid of triangle defined by vertices  $i$ ,  $i+1$ , and  $P$

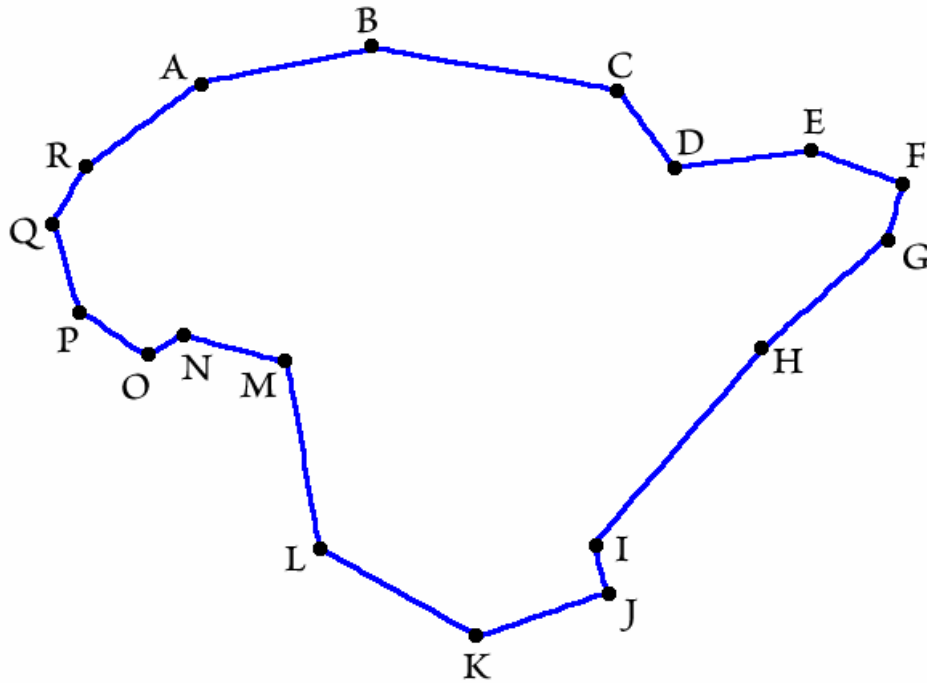
$$\text{such that Centroid } g = \left( \frac{V_{(i)x} + V_{(i+1)x} + P_x}{3}, \frac{V_{(i)y} + V_{(i+1)y} + P_y}{3} \right)$$

For a little more elaborate and visual explanation, the easiest way to calculate the centroid of a polygon is to:

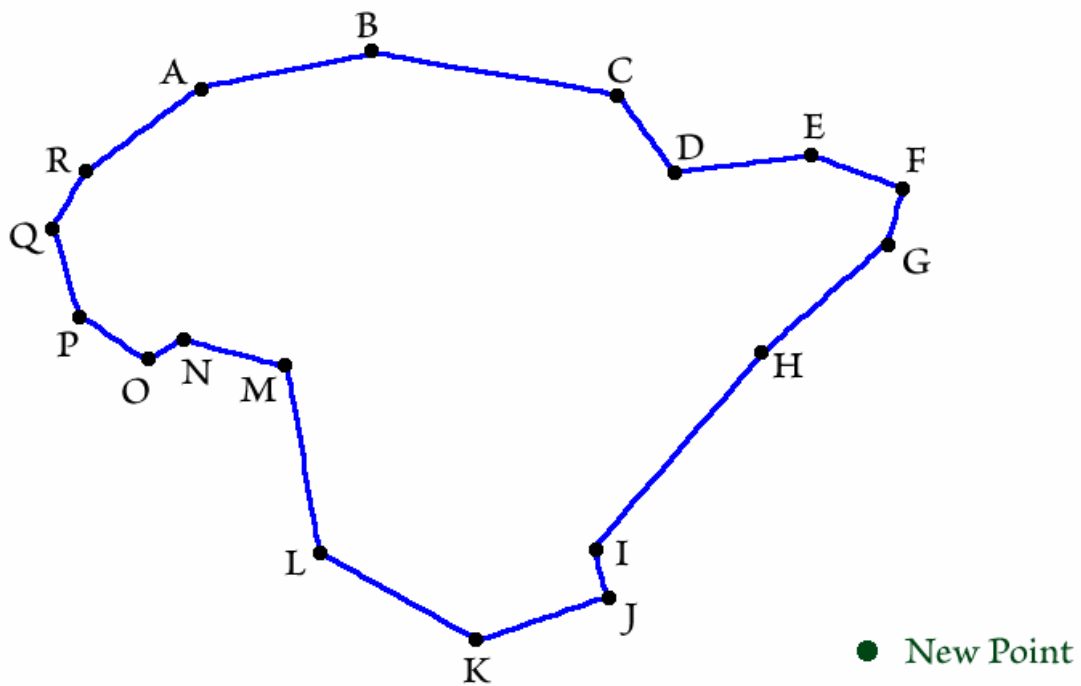
- 1) Break the polygon up into a set of triangles.
- 2) Calculate the area and center of mass of each triangle.
- 3) Take the weighted average of all the triangle centroids, where the triangle centroid is weighted by the triangle area.

There is even an easy way to break up the polygon into triangles. We do not have to worry about generating triangles that occur only within the polygon, because we have a method of calculating the triangle area that gives us a positive value if the vertices are entered in a counterclockwise direction and a negative value if the vertices are entered in a clockwise direction. This means that triangles may be generated outside the polygon boundary, but the area of those triangles are subtracted from the cumulative area, and the final center of mass reflects only the area within the polygon boundary.

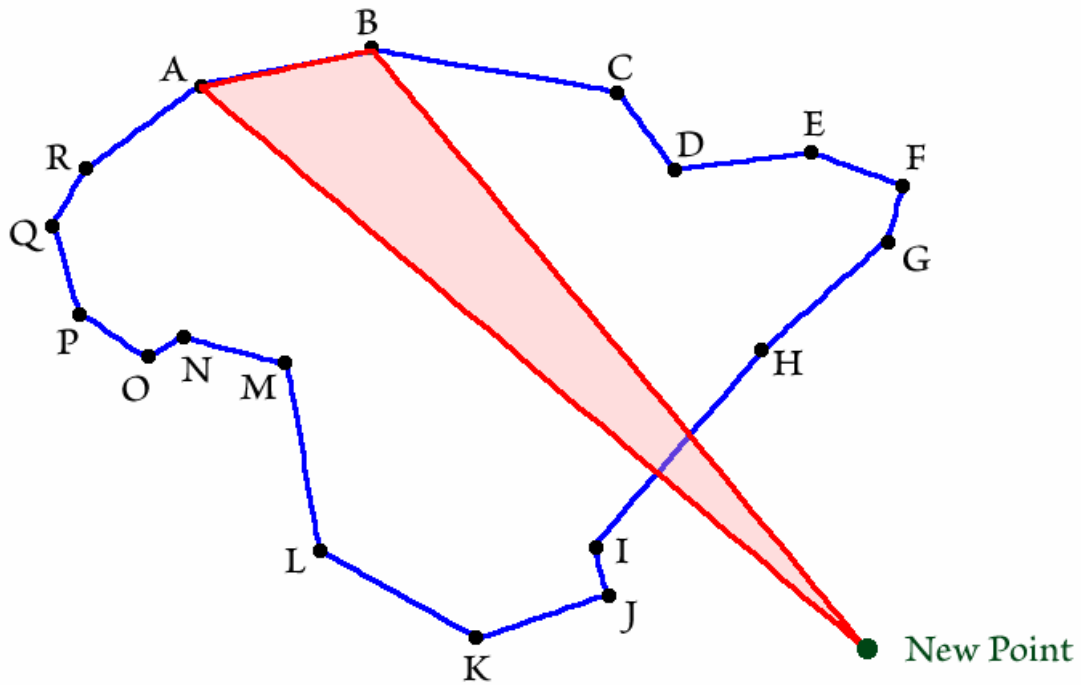
For example, if we had a polygon with 18 vertices labeled A..R:



We start by creating a new point, and then we generate triangles using that new point and all consecutive pairs of vertices. This new point does not even have to be inside the polygon:

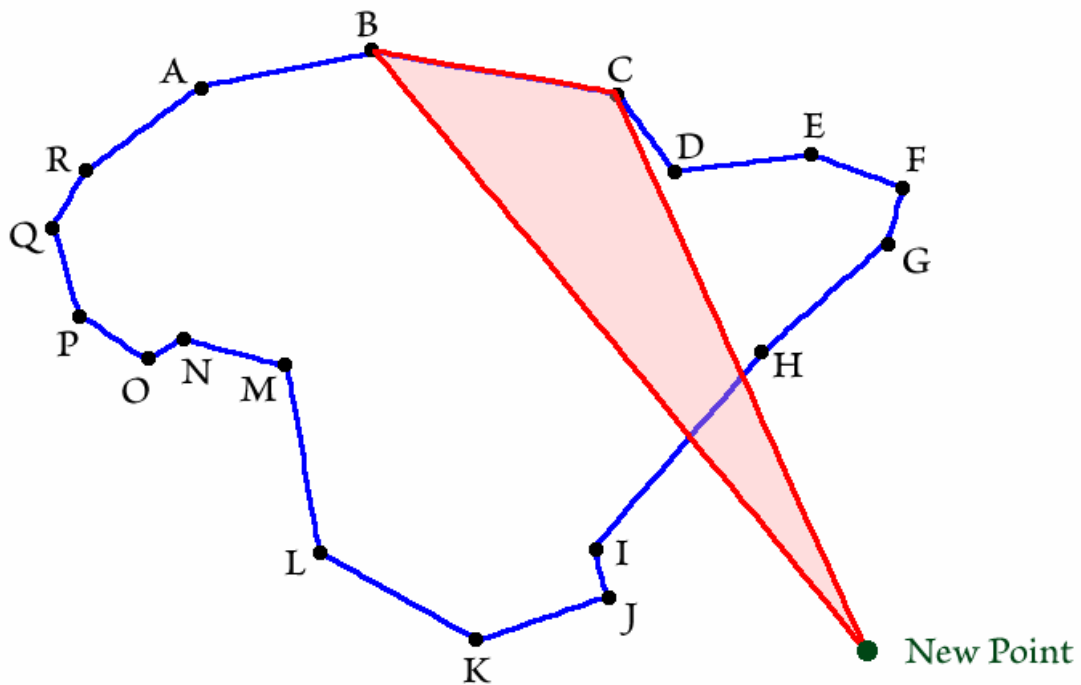


We generate triangles for each consecutive pair of vertices, such that each triangle is composed of vertices  $(V_i, V_{i+1}, \text{New Point})$ . The first triangle includes vertex A, vertex B and the New Point. Note that the vertices are entered in a clockwise direction:

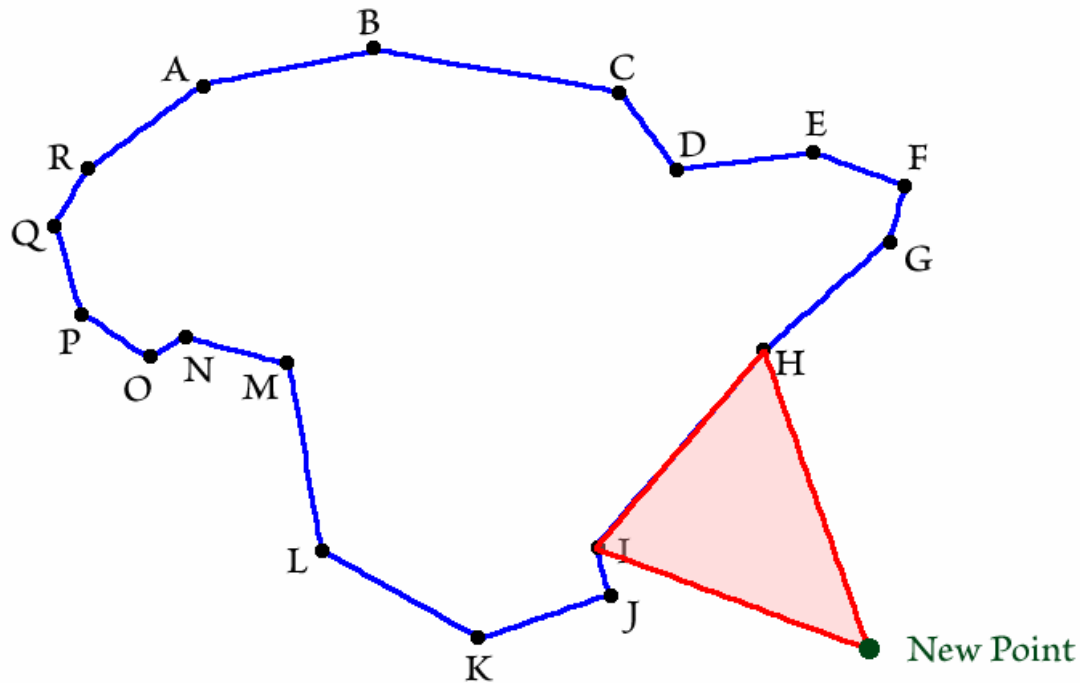


We would calculate the area and center of mass of this triangle and multiply the centroid by the area to get a weighted centroid.

The second triangle would include vertex B, vertex C and the New Point:



Later on, we would generate a triangle from vertex H, vertex I and the New Point:



Note that in this case the vertices are entered in a counterclockwise direction. This is an important factor because this means the sign of the area of  $\triangle HIP$  will be opposite of the signs of the areas of  $\triangle ABP$  and  $\triangle BCP$ , essentially subtracting  $\triangle HIP$  from the cumulative area. Remember that  $\triangle ABP$  and  $\triangle BCP$  include area outside of the polygon, thereby shifting the polygon centroid toward  $P$ . The opposite sign of the area of the counterclockwise triangles clips out all the area external to the polygon, so that the final polygon centroid is affected only the internal polygon area.

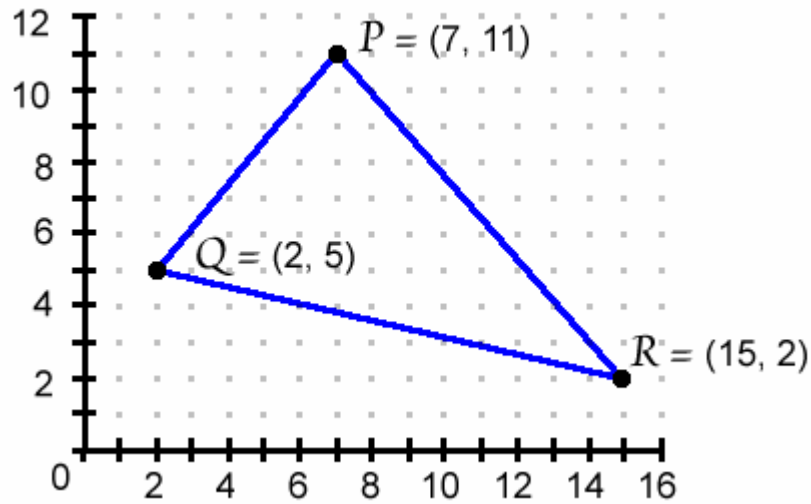
A COMMENT REGARDING THE SIGN OF THE AREA: This is not important to the final position of the centroid, but may be of some interest to those who really look into the math involved. Remember that vertices entered in a clockwise direction generate a negative area value while vertices entered in a counterclockwise direction generate positive area values. Shapefile polygons always have their vertices entered in a clockwise direction. This means that if we take the vertices in order, the internal areas will be negative and the external areas will be positive. The final area of the polygon will be accurate, except that it will be a negative value. If you wanted to use this method to calculate polygon areas, then you would either take the absolute value of the final area or else go through the vertices in reverse order.

The final centroid would be the weighted average of all the triangle centroids, calculated as the sum of the weighted centroids divided by the total polygon area.



## Calculating Triangle Areas and Centroids:

Give a triangle  $\triangle PQR$  with vertices  $P(7,11)$ ,  $Q(2,5)$  and  $R(15,2)$ :



The center of mass of a triangle is easy because it is simply the average of the 3 vertices:

$$\begin{aligned}\text{Center of Mass} &= \left( \frac{P_x + Q_x + R_x}{3}, \frac{P_y + Q_y + R_y}{3} \right) \\ &= \left( \frac{7 + 2 + 15}{3}, \frac{11 + 5 + 2}{3} \right) \\ &= (8, 6)\end{aligned}$$

The area of a triangle  $\triangle PQR$  can be calculated as half the determinant  $|D|$  of the matrix

$$\begin{bmatrix} 1 & P_x & P_y \\ 1 & Q_x & Q_y \\ 1 & R_x & R_y \end{bmatrix}$$

which, given our 3 points, equals

$$\begin{bmatrix} 1 & 7 & 11 \\ 1 & 2 & 5 \\ 1 & 15 & 2 \end{bmatrix}$$

(de Bert et al. 1990, p. 16).

The determinant  $|D|$  of a  $3 \times 3$  matrix  $\begin{bmatrix} A & B & C \\ D & E & F \\ G & H & I \end{bmatrix} = AEI - AFH - BDI + BFG + CDH - CEG$  (Meyer

2000, p. 461). Given our 1<sup>st</sup> column of 1 values, this can be expressed as:

$$|D| \text{ of } \begin{bmatrix} 1 & P_x & P_y \\ 1 & Q_x & Q_y \\ 1 & R_x & R_y \end{bmatrix} = Q_x R_y - Q_y R_x - P_x R_y + P_x Q_y + P_y R_x - P_y Q_x$$

The area of the triangle  $\Delta PQR = \frac{1}{2}|D|$ , so:

$$\begin{aligned}
 \text{Area} &= \frac{1}{2}(Q_x R_y - Q_y R_x - P_x R_y + P_x Q_y + P_y R_x - P_y Q_x) \\
 &= \frac{(Q_x - P_x)(R_y - P_y) - (R_x - P_x)(Q_y - P_y)}{2} \\
 &= \frac{(2-7)(2-11) - (15-7)(5-11)}{2} \\
 &= 46.5
 \end{aligned}$$

NOTE: For those who are not convinced that:

$$\frac{1}{2}(Q_x R_y - Q_y R_x - P_x R_y + P_x Q_y + P_y R_x - P_y Q_x) = \frac{(Q_x - P_x)(R_y - P_y) - (R_x - P_x)(Q_y - P_y)}{2}$$

This is a little easier to prove going the other direction. First, remember from algebra that:

$$(a - b)(c - d) = ac - bc - ad + bd$$

Therefore:

$$\begin{aligned}
 \text{Area} &= \frac{(Q_x - P_x)(R_y - P_y) - (R_x - P_x)(Q_y - P_y)}{2} \\
 &= \frac{1}{2}(Q_x R_y - P_x R_y - Q_x P_y + P_x P_y - (R_x Q_y - P_x Q_y - R_x P_y + P_x P_y)) \\
 &= \frac{1}{2}(Q_x R_y - P_x R_y - Q_x P_y + P_x P_y - R_x Q_y + P_x Q_y + R_x P_y - P_x P_y) \\
 &= \frac{1}{2}(Q_x R_y - P_x R_y - Q_x P_y - R_x Q_y + P_x Q_y + R_x P_y) && (P_x P_y \text{ cancels}) \\
 &= \frac{1}{2}(Q_x R_y - Q_y R_x - P_x R_y + P_x Q_y + P_y R_x - P_y Q_x) && (\text{Just rearrange terms})
 \end{aligned}$$

## Notes for Avenue Programmers:

The script that generates the polygon centroid is named *CenterMass.CalcCenterOfMassArea*. If you send it a polygon, it will return a list containing 2 items:

- 1) The true centroid of the polygon
- 2) The area of the polygon

For example, your code might look like the following:

```
'-----  
CalcData = av.FindScript("CenterMass.CalcCenterOfMassArea")  
For each aPolygon in MyListOfPolygons  
  theData = CalcData.DoIt(aPolygon)  
  theTrueCentroid = theData.Get(0)          ' TRUE CENTER OF MASS  
  theArea = theData.Get(1)                  ' POLYGON AREA  
  theESRICentroid = aPolygon.ReturnCenter   ' ESRI VERSION OF CENTROID  
end  
'-----
```

The script itself is not very complicated. If you would like to insert the code itself into your project, rather than loading this extension, just copy and paste the following code and use the example above to access it:

```
'-----  
' CenterMass.CalcCenterOfMassArea  
' Jenness Enterprises <www.jennessent.com>  
  
' LOOSELY ADAPTED FROM ALGORITHMS IN JOSEPH O'ROURKE (1998):  
' COMPUTATIONAL GEOMETRY IN C, 2ND EDITION, CAMBRIDGE UNIVERSITY PRESS. P. 21  
  
if (self.IsNull) then  
  return {Point.MakeNull, Number.MakeNull}  
else  
  
  theESRICentroid = self.ReturnCenter  
  theCentX = theESRICentroid.GetX  
  theCentY = theESRICentroid.GetY  
  
  thePolygonList = self.Explode  
  
  theAreas = {}  
  theCenters = {}  
  for each aPoly in thePolygonList  
    theVertices = aPoly.AsMultipoint.AsList  
    for each anIndex in 0..(theVertices.Count-2)  
      theP = theVertices.Get(anIndex)  
      theQ = theVertices.Get(anIndex+1)  
  
      theArea = (((theQ.GetX - theP.GetX)*(theCentY - theP.GetY)) -  
                ((theCentX - theP.GetX)*(theQ.GetY - theP.GetY)))/2  
  
      theCenter = (theP + theQ + theESRICentroid)/(3@3)  
  
      theAreas.Add(theArea)  
      theCenters.Add(theCenter)  
    end  
  end  
  
  theCount = theAreas.Count  
  
  theArea = 0  
  theCentroid = 0@0  
  
  for each anIndex in 0..(theCount-1)  
    theSubArea = theAreas.Get(anIndex)  
    theArea = theArea+theSubArea  
    theCentroid = theCentroid+(theCenters.Get(anIndex) * (theSubArea@theSubArea))  
  end  
end  
'-----
```

```
end

theCentroid = theCentroid / (theArea@theArea)

return {theCentroid, theArea.Abs}
end
'-----
```

## References:

- J. O'Rourke. 1998. Computational Geometry in C (2nd Ed.) Cambridge University Press.
- M. de Bert, M. van Kreveld, M. Overmars & O. Schwarzkopf. 1990. Computational Geometry: Algorithms and Applications (2nd ed.). Springer-Verlag.
- Meyer, Carl D. 2000. Matrix Analysis and Applied Linear Algebra. Society for Industrial and Applied Mathematics. Philadelphia, PA, USA.

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Enjoy! Please contact the author if you have problems or find bugs.

Jeff Jenness  
Jenness Enterprises  
3020 N. Schevene Blvd.  
Flagstaff, AZ 86004  
USA

[jeffj@jennessent.com](mailto:jeffj@jennessent.com)  
<http://www.jennessent.com>  
(928) 607-4638

Updates to this extension and an on-line version of this manual are available at

<http://www.jennessent.com/arcview/centermass.htm>

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