## Surface Area and Ratio



## An ArcGIS Extension for Analyzing Raster Elevation Datasets




Jenness Enterprises

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## About the Author

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NAME: Surface Area and Ratio for ArcGIS
Install File: surface_area.exe
Last modified: May 27, 2010
TOPICS: Sphere, geodesic, great circle, surface, area, ratio, elevation, topography, roughness, convolution, convolutedness, rugosity, terrain, geomorphology
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Recommended Citation Format: For those who wish to cite this extension, the author recommends something similar to:
Jenness, J. 2010. Surface Area and Ratio for ArcGIS (surface_area.exe) v. 1.0.138. Jenness Enterprises. Available at: http://www.jennessent.com/arcgis/surface area.htm.

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## Installation

Install the Surface Area and Ratio extension by double-clicking on the file "surface_area.exe" and following the instructions. The installation routine will register the Surface_Area.dll with all the required ArcMap components.
The default install folder for the extension is named "Surface_Area" and is located inside the folder "Program Files\Jennessent". This folder will also include some additional files and this manual.

These tools are installed as an extension in ArcMap, but it is a type of extension that is automatically loaded. You will not see this extension in the "Extensions" dialog available in the ArcGIS "Tools" menu. It is not dependent on any other extensions or any ArcGIS license level.
After installing the extension, you should see the following new toolbar in your map (it may also be embedded in your standard ArcMap toolbars, rather than as a standalone object):


If you do not see this toolbar, then open your "Customize" tool by either:

1) Double-clicking on a blank part of the ArcMap toolbar, or
2) Clicking the "Tools" menu, then "Customize".

In the "Customize" dialog, click the "Toolbars" tab and check the box next to "Graphics and Shapes":


You should now see the Surface Area and Ratio toolbar.

## Uninstalling Surface Area and Ratio

1) Click the Start button.
2) Open your Control Panel.
3) Double-click "Add or Remove Programs".
4) Scroll down to find and select "Graphics and Shapes Tools".
5) Click the "Remove" button and follow the directions.


## Copying and Adding the Tool to Other Toolbars

Because of the way ArcGIS handles toolbars and command buttons, you may add the Surface Area command button to any toolbar you wish.
To do this, open your "Customize" tool by either:

1) Double-clicking on a blank part of the ArcMap toolbar, or
2) Clicking the "Tools" menu, then "Customize".

In the "Customize" dialog, click the "Commands" tab and scroll down to select "Jenness Enterprises":


Finally, simply drag any of the commands out of the Customize dialog up into any of the existing ArcGIS toolbars.

## If Any of the Tools Crash

If a tool crashes, you should see a dialog that tells us what script crashed and where it crashed. I would appreciate it if you could take screenshots of those dialogs and email them to me at jeffj@jennessent.com.

## Description

This extension allows you to generate Surface Area and Surface Ratio rasters from an existing Elevation raster. The cell values for these new rasters reflect the surface area and (surface area) / (planimetric area) ratio for the land area contained within that cell's boundaries. Both Surface Area and Surface Ratio provide useful indices of topographic roughness and convolutedness, and can give a more realistic estimate of the land area available than you can get from the simple planimetric area.


## Background

Landscape area is almost always presented in terms of planimetric area, as if a square kilometer in the Himalayas represents the same amount of land area as a square kilometer in the Nebraska. Predicted home ranges for wildlife species generally use planimetric area even when describing mountain-dwelling species such as mountain goats (Oreamnos americanus) and puma (Felis concolor). But if a species' behavior and population dynamics are a function of available resources, and if those resources are spatially limited, then these resources might be better assessed using surface area of the landscape.

Surface area also is a basis for a useful measure of landscape topographic roughness. The surface area ratio of any particular region on the landscape can be calculated by dividing the surface area of that region by the planimetric area. For example, Bowden et al. (2003) found that ratio estimators of Mexican spotted owl (Strix occidentalis lucida) population size were more precise using a version of this surface area ratio than with planimetric area.
There are a variety of methods in the literature for measuring terrain irregularity. Hobson (1972) described some early computational methods for estimating surface area and discussed the concept of surface area ratios. Beasom (1983) described a method for estimating land surface ruggedness based on the intersections of sample points and contour lines on a contour map, and Jenness (2000) described a similar method based on measuring the density of contour lines in an area. Mandelbrot (1983:29, 112-115) described the concept of a "fractal dimension" in which the dimension of an irregular surface lies between 2 (representing a flat plain) and 3 (representing a surface that goes through every point within a volume). Calculating this fractal dimension can be very challenging computationally, and Polidori et al. (1991), Lam and De Cola (1993) and Lorimer et al. (1994) discussed a variety of methods for estimating the fractal dimension for a landscape.

An estimate of surface area also could be derived from slope and aspect within a cell (Berry 2002), although Hodgson (1995) demonstrated how most slope-aspect algorithms generate values reflecting an area $1.6-2$ times the size of the actual cell. Surface area values derived with this method would, therefore, be unduly influenced by adjacent cells. For those who are interested, however, the method is quick, easy and intuitive. Berry's method can be thought of as simply using the slope to calculate an adjustment factor for the cell planimetric area. If the slope is flat, then the adjustment factor is equal to 1 and therefore the surface area is equal to the planimetric area. As the slope increases, the adjustment factor is equal to:

$$
\text { Adjustment Factor }=\frac{1}{\operatorname{Cos}(\text { Slope })}
$$

where Slope is in Radians, calculated as:

$$
\text { Slope }(\text { Radians })=\frac{[\text { Slope }(\text { Degrees })] \pi}{180}
$$

Therefore, assuming cell size is defined as the length along one edge of the cell, and Slope is measured in degrees, then surface area can be calculated as:

$$
\text { Surface Area }=\frac{c^{2}}{\operatorname{Cos}\left(S\left(\frac{\pi}{180}\right)\right)}
$$

where:

$$
\begin{aligned}
& c=\text { Cell Size } \\
& S=\text { Slope in Degrees }
\end{aligned}
$$

One important factor to note about this method is that, because the cell size is constant for all cells, the final surface area raster is perfectly correlated with the Slope raster. Therefore you wouldn't want to include both Slope and Surface Area in the same statistical model.

## Does not require any additional extensions

This extension uses only standard ArcGIS ArcObjects functions and therefore does not require a license for either Spatial Analyst or 3D Analyst. This extension also calculates surface areas much faster than 3D Analyst, and with as much accuracy. However, because the output is in Raster format rather than Vector, surface areas within polygons cannot be determined as precisely as you can do with a TIN. Surface areas within a polygon are calculated as the sum of the surface area cell values of all those cells whose cell centers lie within the polygon boundaries.
Generating surface area in raster format offers the advantage of allowing you to conduct neighborhood analyses on Surface Area rasters, such that you can generate a raster of statistics describing all cells within a particular range of each cell in the raster. This is useful if you want to conduct analyses at different spatial scales.

## Works on Projected and Unprojected Rasters

Projected and unprojected rasters must be analyzed differently because of the fundamental difference in the measurement units of the coordinate systems. These methods are described later in the manual, but this extension will automatically detect the coordinate system of the raster and use the method appropriate for that coordinate system.

## Current Version:

Version 1.0.131:
July 8, 2009

## General Instructions

1) Load your elevation raster into ArcGIS.
2) Click the Surface Area and Ratio Button to open the "Surface Area Analysis Parameters:" dialog.

3) Select your elevation raster from the listbox of rasters in your map. If you wish to analyze a particular band of a multi-band raster, then click the "All Rasters" tab and select the specific band you wish to analyze.
4) Specify the elevation units of your dataset. The tool is currently written to only accept elevation values in meters or feet because those are by far the most common. Please contact the author (jeffj@jennessent.com) if you need additional elevation unit options.
5) Choose which of three output rasters you wish to generate. You may generate any or all of these in a single analysis.
a. Surface Area: This will produce a raster in which the cell values reflect the topographic surface area within that cell, calculated according to methods described later in this manual.

Note: You have the option to set the areal units for your output. Your options are Square Meters, Hectares, Square Kilometers, Square Feet, Acres and Square Miles. All of these options are available regardless of whether your original elevation units are meters or feet, or whether your original coordinate system was metric, standard
or geographic. The tool will make the appropriate conversion regardless of the original data units.
b. Surface Ratio: This will produce a raster in which the surface area values of each cell are divided by the planimetric area of that cell. These values can potentially range from 1 to infinity, but in practice rarely reach as high as 3 .
c. Flat Area: This is the planimetric area of the cell. If your raster is projected then all cells will have the same value, equal to (cell size) ${ }^{2}$. If your raster is in geographic coordinates (i.e. latitude / longitude), then the cell sizes will increase as you get closer to the equator.

Note: You have the same options for areal units here as you do with the Surface Area raster.
6) Click "OK" to run the analysis. Upon completion, your rasters will be automatically added to your map and you will be shown a report detailing what the tool did.

Report

## -- Analysis Complete --

## Input Parameters:

-- Elevation Raster: elevation_geo
-- Elevation Units: Meters
-- Elevation Projection: GCS_North_American_1983
-- Elevation $\mathrm{X} / \Upsilon$ Units: Decimal Degrees [Cell sizes computed using spherical trigonometry)
*** Spheroid Serni-Major (Equatorial) Axis $=6378137$ meters
$* * *$ Spheroid Serni-Minor (Polar) Axis $=6356752.31414036$ meters

## Outputs:

-- Surface Area Raster: surf_area
Raster Type: ESRI Grid
Workspace: D:TEMP
Area Units: Sq. Meters
-- Surface Ratio Raster: surf_ratio
Raster Type: ESRI Grid
Workspace: D:TTEMP
-- Surface Area Raster: flat_area
Raster Type: ESRI Grid
Workspace: D:TTEMP
Area Units: Acres

Analysis Began: Friday, July 03, 2009 at $8: 05: 15 \mathrm{PM}$ Analysis Complete: Friday, Joly 03, 2009 at $8: 05: 29$ PM Time Elapsed: Time Elapsed: 13 seconds...


## Methods

Methods used by this extension are described in:
Jenness, J. S. 2004. Calculating landscape surface area from digital elevation models. Wildlife Society Bulletin. 32(3):829-839

In summary, this extension calculates surface areas using the following strategy:

- For each cell in the raster, surface areas are based on triangle areas derived from eight triangles
- Each triangle connects the center point of the central cell with the center points of two adjacent cells. These triangles are located in three-dimensional space, so that the area of the triangle represents the true surface area of the space bounded by the three points.
- The triangle area is adjusted so that it only represents the portion of the triangle that overlays the central cell.
- The areas of the eight triangles are summed to produce the total surface area of that cell.
- The surface ratio of the cell is calculated by dividing the surface area of the cell with the planimetric area of the cell.


## Triangulating the Surface

This extension derives surface areas for a cell by using information from the eight cells adjacent to the center cell. For example, given a sample elevation raster:

| 210 | 190 | 170 | 155 | 140 | 135 |
| :--- | :--- | :--- | :--- | :--- | :--- |
| 204 | 183 | 165 | 145 | 125 | 120 |
| 200 | 175 | 160 | 122 | 110 | 100 |
| 208 | 187 | 165 | 150 | 126 | 120 |

this extension would calculate the surface area for the cell with elevation value "183" based on the elevation values of the eight cells surrounding it,

| 210 | 190 | 170 | 155 | 140 | 135 |
| :--- | :--- | :--- | :--- | :--- | :--- |
| 204 | 183 | 165 | 145 | 125 | 120 |
| 200 | 175 | 160 | 122 | 110 | 100 |
| 208 | 187 | 165 | 150 | 126 | 120 |

and the surface area for the "165" cell based on the eight elevation value surrounding it:

| 210 | 190 | 170 | 155 | 140 | 135 |
| :--- | :--- | :--- | :--- | :--- | :--- |
| 204 | 183 | 165 | 145 | 125 | 120 |
| 200 | 175 | 160 | 122 | 110 | 100 |
| 208 | 187 | 165 | 150 | 126 | 120 |

The following example of a Surface Area calculation will be based on the illustration above, where we will calculate the surface area for a cell with an elevation value of 165 , surrounded by cells at $190,170,155,183,145,175,160$ and 122. This raster of elevation values can be pictured In 3dimensional space as a set of adjacent columns, each rising as high as it's specified elevation value.


We take the 3-dimensional centerpoints of each of these 9 cells, and calculate the lengths of the 8 lines that connect the central cell's centerpoint with the center points of the 8 surrounding cells. We then calculate the lengths of the lines that connect each of the 8 surrounding cells with the one right next to it, so that we end up with the lengths of the sides of the 8 triangles that all meet at the center point of the central cell. Using these lengths, we can calculate the areas of each of the triangles.


These lengths are simple to calculate using the Pythagorean Theorem, which states that, in a right triangle, the square of the length of the hypotenuse is equal to the sum of the squares of the other two sides. Thus, for any two points $\mathbf{A}$ and $\mathbf{B}$ :

Pythagorean Theorem:

$$
a^{2}+b^{2}=c^{2} \quad \text { or } \quad c=\sqrt{a^{2}+b^{2}}
$$

where:
$a=$ planimetric distance from Point $A$ to Point $B$
$b=$ difference in elevation between Point $A$ and Point $B$
$c=$ True surface distance from Point A to Point $B$

" b " is easy to calculate, since it is simply the difference between the two cell values. "a" is easy to get for the cells directly to the North, East, South and West, since it is simply the value of the cell size. For the diagonals, we use the Pythagorean theorem again and calculate $a=\sqrt{2(\text { cell size })^{2}}$.

However, since we're only concerned with the surface area within each cell, we cut all the triangle lengths in half. Now when we calculate the area of a triangle based on the lengths of three sides, the three sides will represent the portion of the triangle that lies within the cell boundaries.


This action is justified based on the Side-Angle-Side similarity criterion for similar triangles (Euclid 1956:204), which states that "If two triangles have one angle equal to one angle and the sides about the equal angles proportional, the triangles will be equiangular and will have those angles equal which the corresponding sides subtend". Each original triangle is "similar" to its corresponding clipped triangle because the 2 sides extending from the center cell in the original triangle are exactly twice as long as the respective sides in the clipped triangle, and the angles defined by these 2 sides are the same in each triangle. Therefore, the third side of the clipped triangle must be exactly half as long as the corresponding side of the original triangle.

Technical Note: The above description only describes the logic used to derive the surface area. The extension is not actually connecting any lines or even using any vector data at all. Rather, the extension examines each cell individually, generating the eight triangle areas in memory based on the 8 cells surrounding it. Finally, the extension adds those triangle areas to produce the final surface area value for that cell.

## Calculating the Area of a Triangle from the Lengths of the Three Sides

This extension uses the Triangle half-perimeter formula to calculate triangle areas, attributed to Heron of Alexandria and Archimedes (see also Abramowitz and Stegun [1972, p. 79]):

$$
\begin{aligned}
\text { Triangle half-perimeter } & =S=\frac{a+b+c}{2} \\
\text { Area } & =\sqrt{S(S-a)(S-b)(S-c)}
\end{aligned}
$$

## Modification for Geographic Coordinates

Latitude and longitude coordinate values present a special challenge to these calculations. Latitude/Longitude values are polar coordinates (i.e. they reflect angular units) rather than projected Cartesian coordinates (i.e. $X, Y$ units), and the methods used to calculate area from Cartesian coordinates are not appropriate for polar coordinates. A triangle area reported in "square degrees" is completely nonsensical.

Furthermore, even though geographic raster cells may look square in a geographically projected map window, they most definitely are not square as they lay on the surface of the earth. For example, given a very coarse raster covering the entire planet with $15^{\circ}$ cell sizes:


A change in $15^{\circ}$ latitude represents approximately the same distance anywhere on the surface of the planet. However, the distance represented by $15^{\circ}$ longitude depends on your latitude. On the equator, $15^{\circ}$ longitude covers the same distance as $15^{\circ}$ latitude. At the poles, $15^{\circ}$ longitude covers no distance at all. The squares in the map above get progressively narrower as you approach the poles, and they are actually triangles at the poles themselves.


We can still use our basic methods described above, but we cannot use the original latitudes and longitudes of the cell centers. We can, however, convert these coordinates to 3-dimensional Cartesian coordinates.

Converting between Latitude / Longitude and Cartesian Coordinates
Note: The following section is adapted from the manual for the author's Tools for Graphics and Shapes extension, available at http://www.jennessent.com/arcgis/shapes_graphics.htm.

This tool calculates distances between cell centerpoints as they lay on the spheroid, rather than as they lay on the sphere. The shape of the earth is modeled better with a spheroid than a sphere so the calculations are slightly more accurate using equations designed for a spheroid.

However, the equations are easier to understand if they are introduced in their simplest form. Therefore this section begins by explaining the conversion methods on the sphere, and then shows how the equations are modified to handle the spheroid.
Latitude / Longitude to Cartesian Coordinate on the Sphere:
Given a Latitude and Longitude in Degrees, calculate $\mathrm{X}, \mathrm{Y}$ and Z as follows:

$$
\begin{aligned}
& \mathrm{X}=R \sin \phi \cos \theta \\
& \mathrm{Y}=R \sin \phi \sin \theta \\
& \mathrm{Z}=R \cos \phi
\end{aligned}
$$

where:

$$
\begin{aligned}
& \phi=\frac{(90-\text { Latitude }) \pi}{180}=\text { Angle from North Pole down to Latitude, in Radians } \\
& \theta=\frac{(\text { Longitude }) \pi}{180}=\text { Angle from Greenwich to Longitude, in Radians } \\
& R=\text { Radius of Sphere }
\end{aligned}
$$

For example, a point at $30^{\circ}$ Latitude and $45^{\circ}$ Longitude would be converted to $\phi=1.05$ radians and $\theta=0.79$ radians:



Note: Be careful with the terms $\phi$ and $\theta$ ! This document treats $\phi$ as the angle south from the North Pole, and $\theta$ as the angle east from Greenwich. However, the literature is not consistent with these two terms! Much of the literature uses these exact two symbols, but for the other angle (i.e. $\phi$ for east/west and $\theta$ for north/south).

Using the formula above, calculate $\mathrm{X}, \mathrm{Y}$ and Z from $\phi$ and $\theta$.

$$
\begin{aligned}
& \mathrm{X}=R \sin \phi \cos \theta=R \sin (1.05) \cos (0.79)=0.61 R \\
& \mathrm{Y}=R \sin \phi \sin \theta=R \sin (1.05) \sin (0.79)=0.61 R \\
& \mathrm{Z}=R \cos \phi=R \cos (0.79)=0.5 R
\end{aligned}
$$



Note: As with $\phi$ and $\theta$, the literature is inconsistent regarding how to write the $\mathrm{X}, \mathrm{Y}$ and Z coordinates of a 3 -dimensional point. If a point is located at $X, Y, Z$, some sources will write the coordinates as ( $\mathrm{X}, \mathrm{Y}, \mathrm{Z}$ ) while other sources will write it as ( $\mathrm{Z}, \mathrm{X}, \mathrm{Y}$ ). If you write coordinates in either of these formats, please identify which format you are using!

Latitude / Longitude to Cartesian Coordinates on the Spheroid:
As illustrated in Chapter 2 of lliffe (2000), the formulas above can be adapted to oblate spheroids (such as a planet). If you refer to lliffe's text, please be aware that his formulae define $\phi$ as Latitude, rather than as ( 90 -Latitude). The formulae below are modified from lliffe's formulae to make them consistent with the conversion formulae for the sphere above.

Given a Latitude and Longitude in Degrees, Spheroid Equatorial and Polar Axes, and an optional height above the spheroid, calculate $X, Y$ and $Z$ as follows:

$$
\begin{aligned}
& \mathrm{X}=(v+h) \sin \phi \cos \theta \\
& \mathrm{Y}=(v+h) \sin \phi \sin \theta \\
& \mathrm{Z}=\left(\left(1-e^{2}\right) v+h\right) \cos \phi
\end{aligned}
$$

where:
$\phi=\frac{(90-\text { Latitude }) \pi}{180}=$ Angle from North Pole down to Latitude, in Radians
$\theta=\frac{(\text { Longitude }) \pi}{180}=$ Angle from Greenwich to Longitude, in Radians
$a=$ Spheroid equatorial radius (i.e. semi-major axis)
$b=$ Spheroid polar radius (i.e. semi-minor axis)
$e^{2}=$ Spheroid eccentricity (squared) $=\frac{a^{2}-b^{2}}{a^{2}}$
$v=\frac{a}{\sqrt{1-e^{2} \cos ^{2} \phi}}$
$h=$ Height above the spheroid

## Creating Triangles on the Spheroid

Once the latitude and longitude coordinates are calculated, then the process is very similar to that for projected rasters. The main difference is that the cell centerpoints extend out from the curved surface of a planet rather than from a projected plane, but this makes no difference to the mathematics.
The surface area for a cell is based on the elevation in that cell, plus the elevations in the 8 cells around it.


3-dimensional triangles are generated (conceptually, at least; no actual vector objects are used in the calculations) from the centerpoints of the cells.


Triangles are clipped to the region covered by the central cell.


Technical Note: When analyzing a geographic raster, this tool treats the cell as a flat plane as illustrated in the images above. It does not consider the surface of the cell to be curved.
Planimetric area is therefore equal to the area of the trapezoidal boundary of the cell, and surface area will always be $\geq$ planimetric area.

## Accuracy Analysis: TIN vs. Raster Surface Area Comparison

Note: This analysis was actually done using the ArcView 3.x version of the Surface Areas and Ratios extension (see http://www.jennessent.com/arcview/surface areas.htm). The author has confirmed that the outputs of these extensions are identical, so the discussion below is simply adapted from the manual for the previous extension.

## Summary

I compared the output of the Surface Area and Surface Ratio rasters derived by methods used in this extension with surface areas and ratios derived from a TIN-based analysis using the ESRI's 3D Analyst extension (ESRI 2000) and the author's Surface Tools for ArcView 3.x. extension (Jenness 2001). I computed the surface areas and ratios within 650 circular polygons ranging in size from 100m radius (approximately 3 raster cells) to 20,000m radius (approximately 146,454 raster cells).

Surface Ratios: I found that surface ratios correlated very well at all scales. The lowest correlation between surface ratio values was 0.993 (circle radius $=100$ ). Raster-based values tended to be very close to TIN-based values, never differing more than $1 \%$ from their corresponding TIN-based value. In almost all cases, both accuracy and precision increased as the size of the circle increased, so that the circles with the greatest numbers of raster cells tended to be the most accurate and precise.
Surface Areas: Surface area values were extremely close between TIN-based methods and these raster-based methods, with correlations ranging from 0.993 to 1.000. Raster-based values tended to be higher than their TIN-based counterparts at low cell counts (<264), but accuracy greatly increased at cell counts higher than 234 (circle radius $=800 \mathrm{~m}$ ). For polygons with $>=$ 264 cells, raster-based values were almost identical to TIN-based values and both accuracy and precision were nearly perfect.
The overall accuracy of this method, based on several additional analyses, can be also reviewed in:

Jenness, J. S. 2004. Calculating landscape surface area from digital elevation models. Wildlife
Society Bulletin. 32(3):829-839

## Methods

I tested the output accuracy of this extension by generating surface areas and ratios for 650 circular polygons in 7 different sizes and comparing results between a TIN-based method and the raster-based method used by this extension. These 650 polygons were randomly distributed across an elevation raster and all the polygons were completely within the elevation raster boundaries (i.e. none of the polygons overlaid any "No Data" areas). For a control group, I used the Surface Tools extension (Jenness 2001) to calculate what I assume to be the most accurate surface area and surface ratio values possible from the elevation data. I used a Z-factor $=0$ (meaning that the new TIN will go through all the elevation points and therefore be as complex as possible. See Surface Tools manual for an explanation).

I then generated surface areas and ratios using the ArcView 3.x version of this extension. I compared the raster-based surface areas with the TIN-based surface areas for each of the 650 circular polygons using scatterplots, correlation statistics, and descriptive statistics. I used SPSS 9.0 to generate the statistics.

Polygon radii are in meters and rasters had a cell size of approximately 92.6 meters. The means, standard deviations and ranges reflect the ratios of (raster-based surface area) divided by (TINbased surface area), or the ratio of (raster-based surface ratio) divided by (TIN-based surface ratio).

## Results

In all cases, surface ratio values were very close between the TIN- and raster-based calculations. Accuracy and precision increased with higher cell counts, but even in the worst case the raster-based calculation was only $1 \%$ different than the TIN-based calculation.

Surface area values were always highly correlated, but raster-based calculations tended to be higher than TIN-based calculations at low cell counts. Surface area values for the 100 m circles, with 4 raster cells each, tended to be over $9 \%$ higher when calculated from the raster. Rasterbased surface areas for circles with up to 60 cells tended to be about $2.5 \%$ higher than their corresponding TIN-based surface areas.

Surface areas for circles with cell counts $>=234$ tended to be almost identical between the rasterand TIN-based calculations, and correlations were almost perfect.

Table 1: Surface Area Comparisons - TIN vs. Raster-Based Output

| Circle <br> Radius | n | Avg. \# <br> Raster Cells <br> in Polygon | Pearson <br> Correlation | Surface Area Comparison |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  | (RASTER Surface Area) / (TIN Surface Area) |  |  |  |  |
| 100 | 100 | 4 | 0.993 | mean | sd | range |
| 200 | 100 | 15 | 0.996 | 1.0937 | 0.0039 | $1.08-1.11$ |
| 400 | 100 | 60 | 0.999 | 1.0249 | 0.0022 | $1.02-1.04$ |
| 800 | 100 | 234 | 1.000 | 0.9991 | 0.0011 | $1.02-1.03$ |
| 1,000 | 100 | 369 | 1.000 | 1.0084 | 0.0004 | $1.00-1.00$ |
| 2,000 | 100 | 1,465 | 1.000 | 1.0008 | 0.0003 | $1.01-1.01$ |
| 20,000 | 50 | 146,361 | 1.000 | 0.9996 | 0.0003 | $1.00-1.00-1.00$ |

Table 2: Surface Ratio Comparisons - TIN vs. Raster-Based Output

| Circle <br> Radius | n | Avg. \# <br> Raster Cells <br> in Polygon | Pearson <br> Correlation | (RASTER Surface Ratio) / (TIN Surface Ratio) |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  | 4 | 0.993 | mean | sd | range |
| 100 | 100 | 4 | 0.996 | 1.0004 | 0.0036 | $0.99-1.01$ |
| 200 | 100 | 15 | 0.999 | 0.9999 | 0.0021 | $1.00-1.01$ |
| 400 | 100 | 60 | 1.000 | 0.9998 | 0.0011 | $0.99-1.00$ |
| 800 | 100 | 234 | 1.000 | 0.9998 | 0.0003 | $1.00-1.00$ |
| 1,000 | 100 | 369 | 1.000 | 0.9998 | 0.0003 | $1.00-1.00-1.00$ |
| 2,000 | 100 | 1,465 | 1.000 | 0.9995 | 0.0002 | $1.00-1.00$ |
| 20,000 | 50 | 146,361 |  |  |  |  |

## Table 6: TIN vs. Raster-Based Comparison Scatterplots

## Circle Data

Surface Area Scatterplots

TIN and GRID Surface Areas
100m Radius Circles


TIN and GRID Surface Areas
200m Radius Circles


Surface Ratio Scatterplots

TIN and GRID Surface Ratios


TIN and GRID Surface Ratios
200 m Radius Circles


TIN and GRID Surface Areas


TIN and GRID Surface Areas


TIN and GRID Surface Ratios

$n=100 ; 60$ grid cells per circle

TIN and GRID Surface Ratios


## TIN and GRID Surface Areas



TIN and GRID Surface Areas


TIN and GRID Surface Ratios


TIN and GRID Surface Areas


TIN and GRID Surface Areas
20,000m Radius Circles
radius = 20,000 146,361 cells $\mathrm{n}=50$


TIN and GRID Surface Ratios
20,000m Radius Circles


## Discussion

This extension appears to generate accurate surface area and surface ratio rasters. A cell value in a surface area raster appears to accurately reflect the surface area within the boundaries of that cell.
Surface areas within polygons can be accurately measured with this method provided the polygon contains a sufficient number of cells. With the circular polygons used in this analysis, accuracy and precision tended to be very high when cell counts equaled or exceeded 264.

Cumulative surface area and ratio accuracy within polygons is variable when the polygon is small, although most of this variation is likely due to the fact that identically-sized circles randomly placed on a raster will not generally have the same number of cells in each circle. A cell is considered to be included in a circle if that cell's center is located within the circle boundary, and slight deviations in position can cause large numbers of cells to be included or excluded. Accuracy is consistently low in cases of low cell counts, with raster-based values tending to be significantly higher than TIN-based values.

## Recommendations

I feel that this extension does produce reliable surface area and surface ratio values from elevation rasters. Summary statistics within polygons is most accurate when the polygon includes a large number of cells (>=264), and accuracy is variable below that number.

## Updates

## Version 1.0

- Build 1.0.130 (July 7, 2009): Initial Release
- Build 1.0.131 (July 8, 2009): Fixed web site address in "About" dialog.
- Build 1.0.137 (April 8, 2010): Wrote a work-around for an ArcObjects bug in which the rasters would not be correctly written to the hard drive, even when they showed correctly in the map.
- Build 1.0.138 (May 27, 2010): Removed some code that generates a series of message boxes during the analysis.


## References

Abramowitz, M., and I. A. Stegun. 1972. Handbook of mathematical functions with formulas, graphs and mathematical tables. Dover Publications, Inc. New York, New York, USA.
Beasom, S. L. 1983. A technique for assessing land surface ruggedness. Journal of Wildlife Management. 47: 1163-1166.

Berry, J. K. 2002. Use surface area for realistic calculations. Geoworld 15(9): 20-1.
Bowden, D. C., G. C. White, A. B. Franklin, and J. L. Ganey. 2003. Estimating population size with correlated sampling unit estimates. Journal of Wildlife Management 67: 1-10
ESRI. 3D Analyst - ArcView Extension. Review available at http://www.esri.com/software/arcview/extensions/3dext.html.

Euclid. 1956. The thirteen books of Euclid's elements: translated with introduction and commentary by Sir Thomas L. Heath. Volume 2 (Books III-IX). Second Edition Unabridged. Dover Publications, New York, New York, USA.

Hobson, R. D. 1972. Chapter 8 - surface roughness in topography: quantitative approach. Pages 221-245 in R. J. Chorley, editor. Spatial analysis in geomorphology. Harper \& Row, New York, New York, USA.

Hodgson, M. E. 1995. What cell size does the computed slope/aspect angle represent? Photogrammetric Engineering \& Remote Sensing 61: 513-517.
lliffe, Jonathan. 2000. Datums and Map Projections for Remote Sensing, GIS and Surveying. Whittles Publishing, Scotland, UK. 150 pp.
Jenness, J. 2000. The effects of fire on Mexican spotted owls in Arizona and New Mexico. Thesis, Northern Arizona University, Flagstaff, Arizona, USA.
Jenness, Jeff S. 2001. Surface Tools for Points, Lines and Polygons v. 1.3-ArcView Extension. Jenness Enterprises. Available for review and download at http://www.jennessent.com/arcview/surface_tools.htm.
Jenness, Jeff S. In Press. Calculating landscape surface area from digital elevation models. Wildlife Society Bulletin.

Lam, N. S. N., and L. DeCola. 1993. Fractals in Geography. PTR Prentice-Hall, Englewood Cliffs, New Jersey, USA.

Lorimer, N. D., R. G. Haight, and R. A. Leary. 1994. The fractal forest: fractal geometry and applications in forest science. United States Department of Agriculture Forest Service, North Central Forest Experiment Station, General Technical Report; NC-170. St. Paul, Minnesota, USA.

Mandelbrot, B. B. 1983. The fractal geometry of nature. W. H. Freeman and Company, New York, New York, USA.
Polidori, L., J. Chorowicz, and R. Guillande. 1991. Description of terrain as a fractal surface, and application to digital elevation model quality assessment. Photogrammetric Engineering \& Remote Sensing 57: 1329-1332.

SPSS, Inc. 1998. SPSS For Windows. Version 9.0. 444 North Michigan Avenue, Chicago, IL, USA, 60611.

