Surface Areas and Ratios from DEMs

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These functions are available in the ArcGIS extension "Surface Area and Ratio"

All formulae and references are described in full in the manual

Available for free download at http://www.jennessent.com/arcgis/surface_area.htm

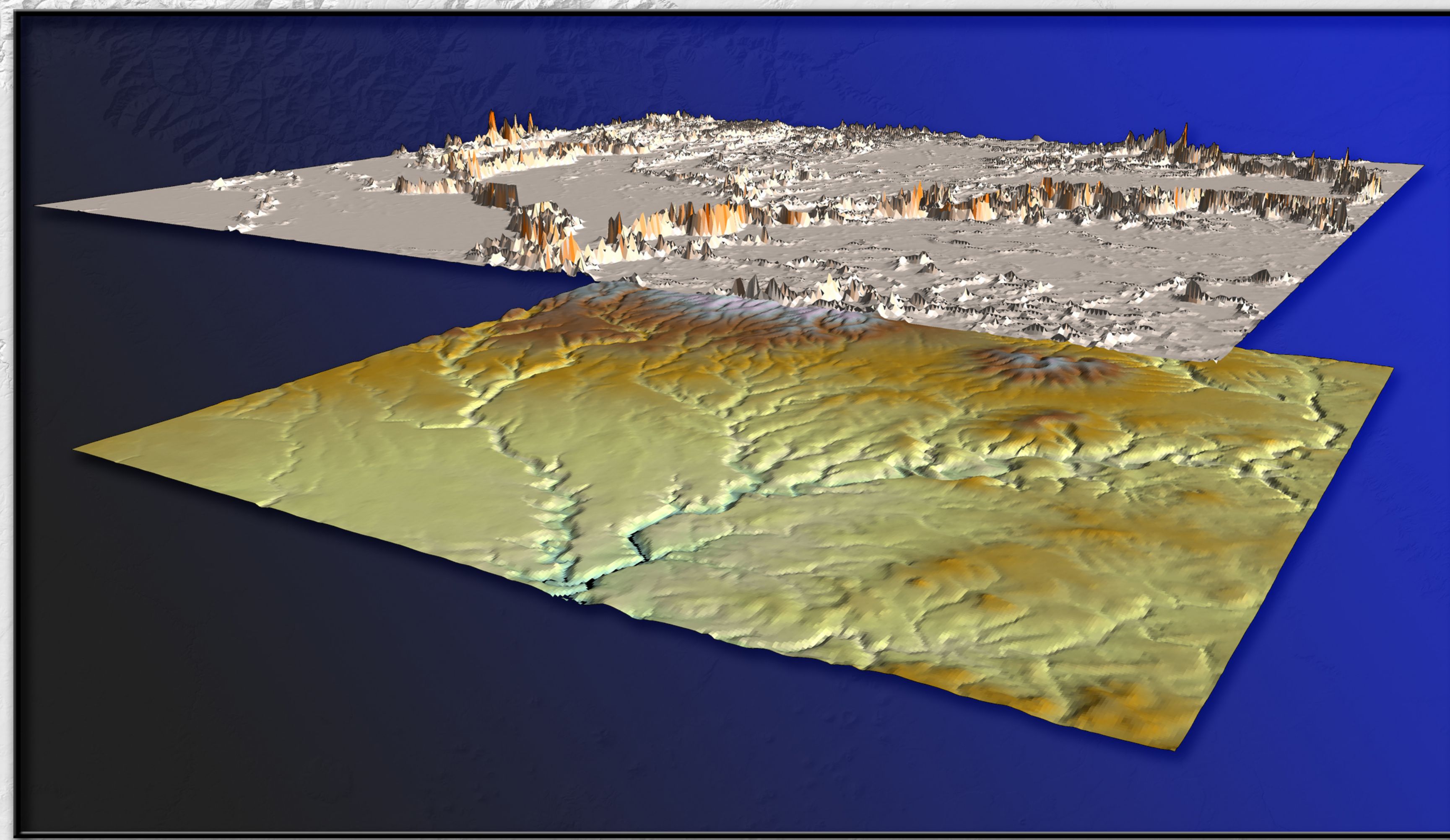
Introduction

The natural landscape is a complex surface divided by canyons and valleys and highlighted with mountain ranges. Landscape area, however, is almost always presented in terms of planimetric area, as if a square kilometer in the Himalayan Mountains represents the same amount of land area as a square kilometer of farmland in Nebraska.

In the field of wildlife biology, predicted ranges for wildlife species generally use planimetric area even when we study mountain-dwelling species such as mountain goats and cougars. But if we believe that a species' behavior and population dynamics are a function of the resources available to that species, and if these resources are spatially limited, then I suggest that we look at the land and resources in terms of the true surface area of the landscape.

Here I demonstrate a reasonably intuitive and straightforward method for calculating surface area from Digital Elevation Models (DEMs), which are comprised of a grid of cells where each cell contains an elevation value. These DEMs are widely and freely available from various sites on the Internet.

The surface area raster can then be divided by a raster of planimetric area to create a raster of surface ratio, providing a useful index of topographic roughness.



Methods

This method derives surface areas for a cell using elevation information from the eight cells adjacent to that cell. For example, given a sample elevation grid (Fig. 1), this method would calculate the surface area for the cell with elevation value "183" based on the elevation values of that cell plus the eight cells surrounding it (Fig. 1a) and the surface area for the "165" cell based on that cell plus the eight elevation values surrounding it (Fig. 1b). We can picture the cell with elevation value "165" and its surrounding elevation values in 3-dimensional space as a set of adjacent columns, each rising as high as it's specified elevation value (Fig. 2).

We take the 3-dimensional centerpoints of each of these 9 cells and calculate the 3-dimensional lengths of the 8 lines that connect the central cell's centerpoint with the center points of the 8 surrounding cells. We then calculate the 3-dimensional lengths of the lines that connect each of the 8 surrounding cells with the ones adjacent to it, so that we end up with the lengths of the sides of the 8 three-dimensional triangles that all meet at the center point of the central cell (Fig. 3).

Finally we clip those triangles to the region of the central cell (Fig. 4). The surface area is defined as the sum of the areas of the 8 3-dimensional triangles.

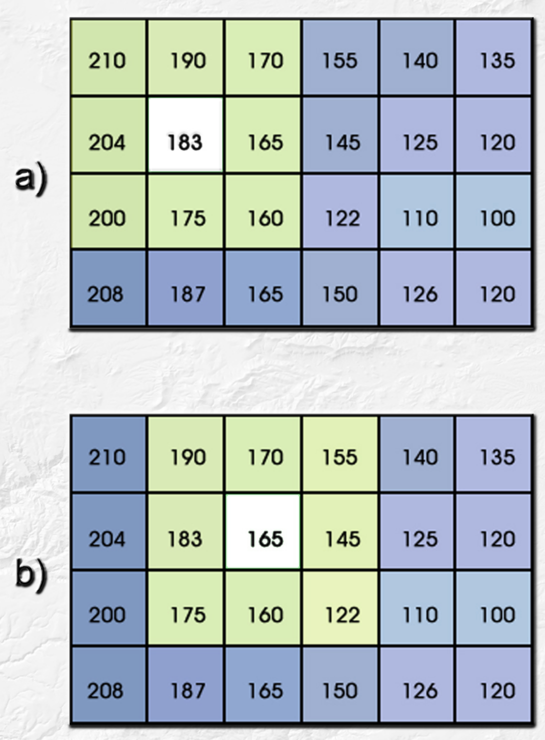


Figure 1: The surface area of each cell is calculated based on the elevation values of that cell plus the eight surrounding cells.

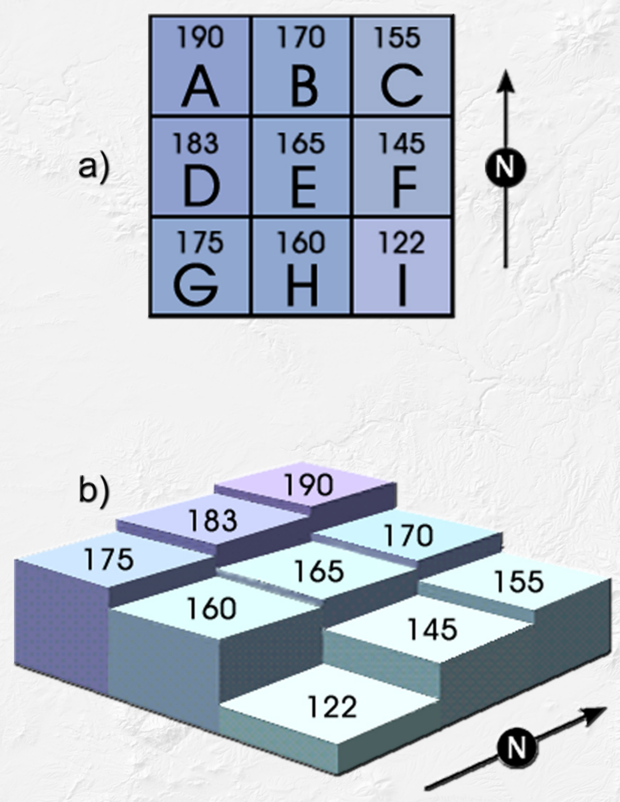


Figure 2: The cells may be pictured 3-dimensionally as a set of adjacent columns.

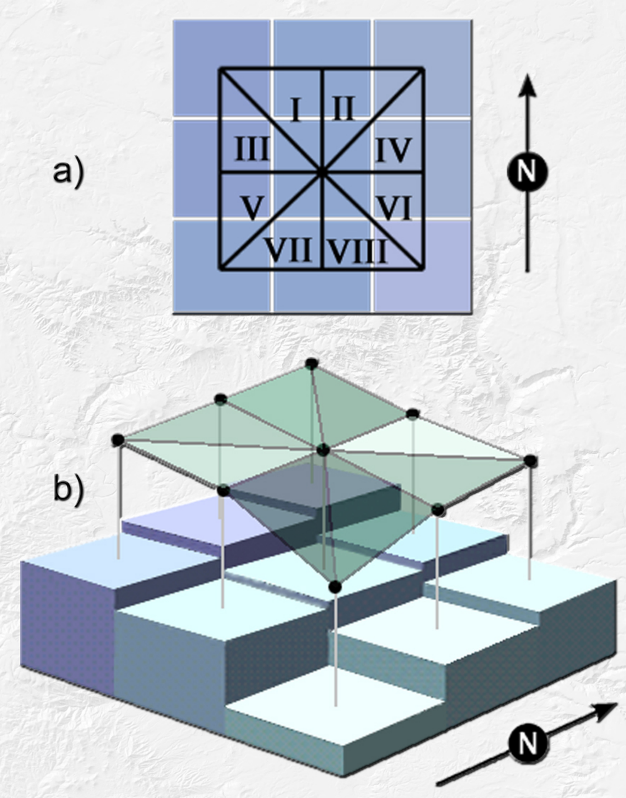


Figure 3: Surface area is calculated by adding the areas of the 8 triangles connecting the center of the central cell with the centers of the eight surrounding cells.

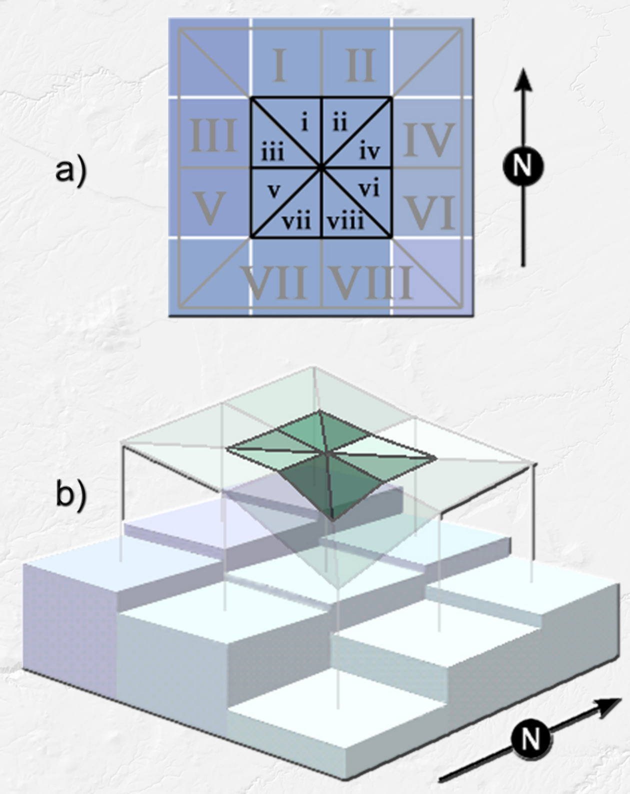
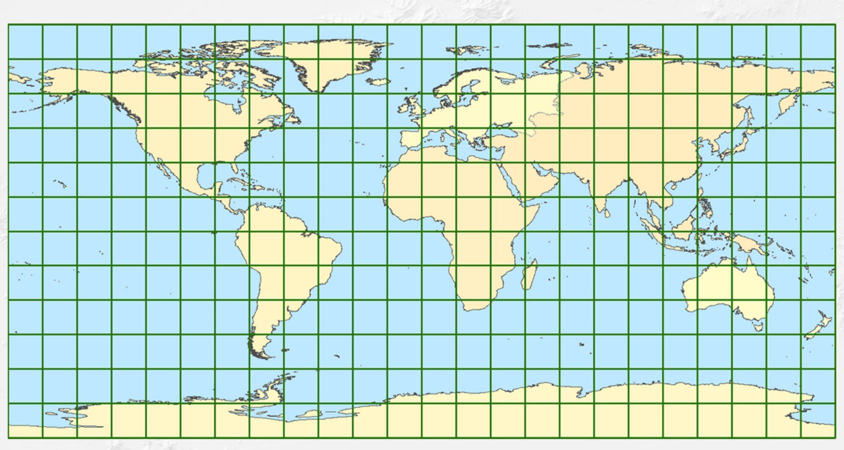


Figure 4: The triangles must be clipped to the central cell boundary so that the area only represents the region covered by that cell.

Modification for Geographic Spatial Reference

Latitude and longitude coordinate values present a special challenge to these calculations. Latitude/Longitude values are polar coordinates (i.e. they reflect angular units) rather than projected Cartesian coordinates (i.e. X,Y units), and the methods used to calculate area from Cartesian coordinates are not appropriate for polar coordinates. A triangle area reported in "square degrees" is completely nonsensical.

Furthermore, even though geographic raster cells may look square in a geographically projected map window, they most definitely are not square as they lay on the surface of the earth. For example, given a very coarse raster covering the entire planet with 15° cell sizes:



A change in 15° latitude represents approximately the same distance anywhere on the surface of the planet. However, the distance represented by 15° longitude depends on your latitude. On the equator, 15° longitude covers the same distance as 15° latitude. At the poles, 15° longitude covers no distance at all. The squares in the map above get progressively narrower as you approach the poles, and they are actually triangles at the poles themselves.

To solve this problem, simply convert the latitude / longitude coordinates to a 3-dimensional Cartesian system before attempting to calculate areas and distances.

$$X = (\nu + h)\sin\phi\cos\theta \qquad Y = (\nu + h)\sin\phi\sin\theta \qquad Z = \left(\left(1 - e^2\right)\nu + h\right)\cos\phi$$

where:

$\phi = \frac{(90 - \text{Latitude})\pi}{180}$ = Angle from North Pole down to Latitude, in Radians

$\theta = \frac{(\text{Longitude})\pi}{180}$ = Angle from Greenwich to Longitude, in Radians

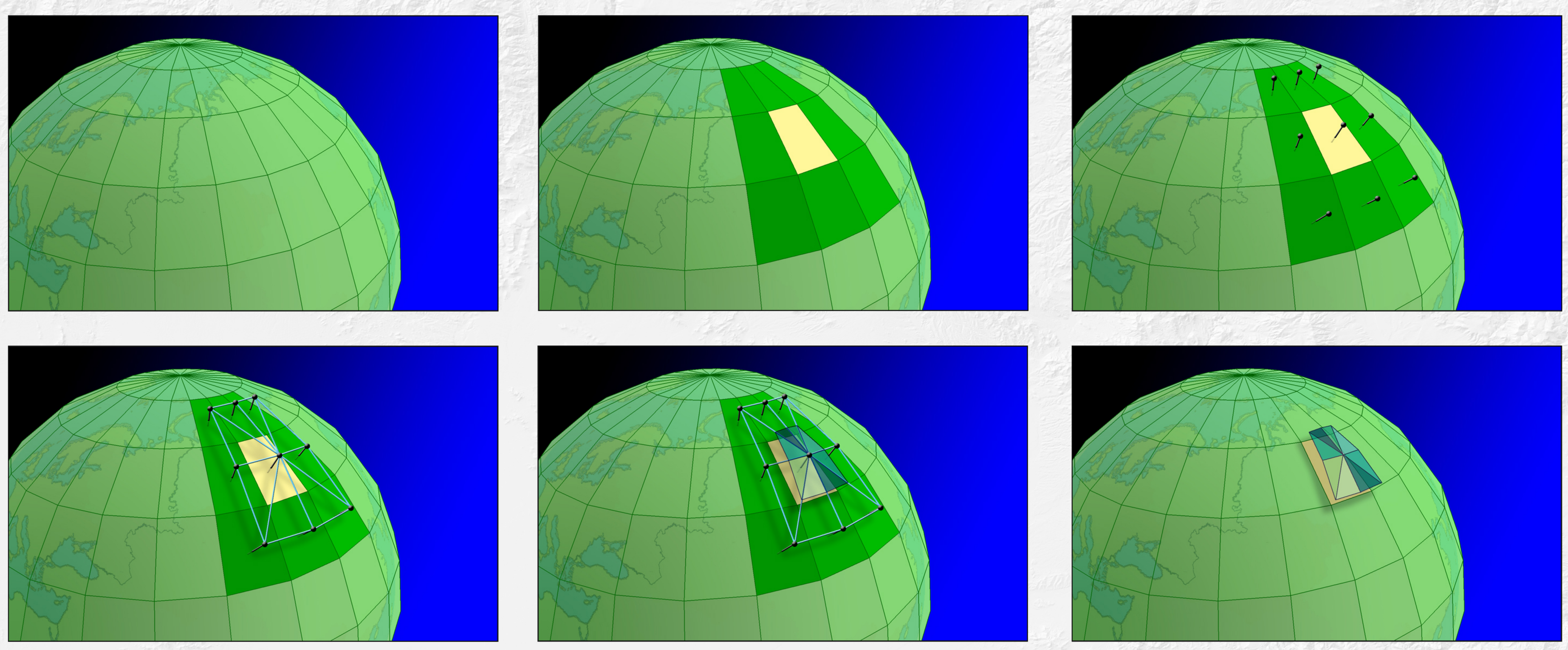
a = Spheroid equatorial radius (i.e. semi-major axis)

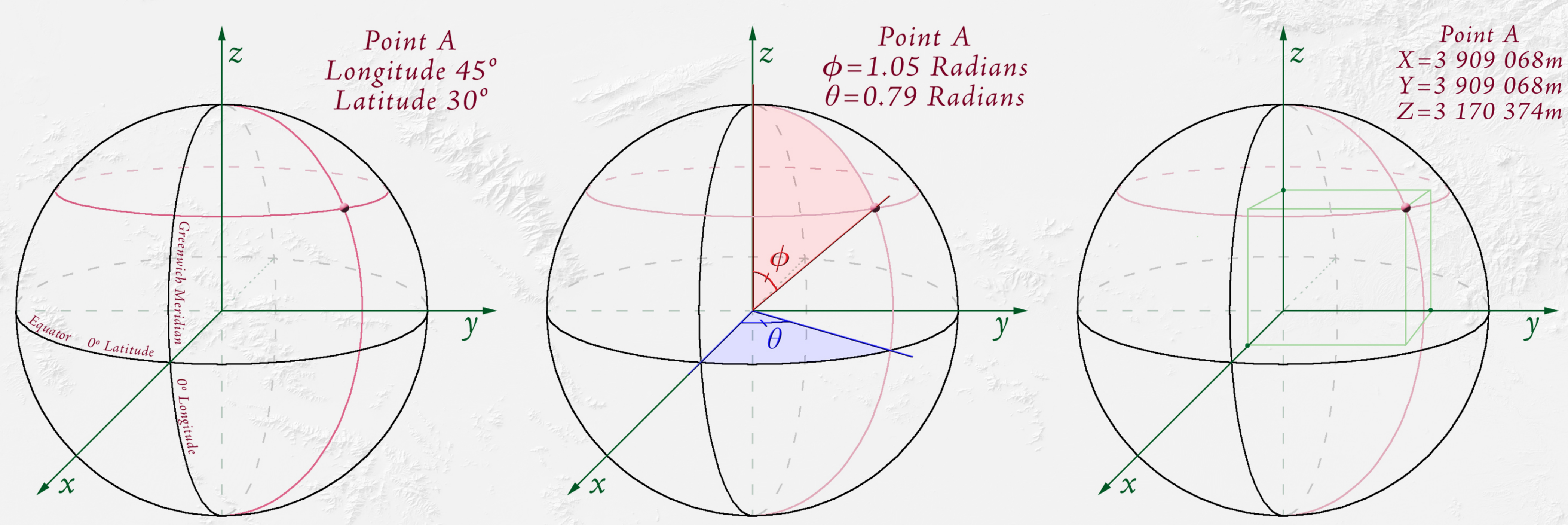
b = Spheroid polar radius (i.e. semi-minor axis)

e^2 = Spheroid eccentricity (squared) = $\frac{a^2 - b^2}{a^2}$

$\nu = \frac{a}{\sqrt{1 - e^2\cos^2\phi}}$

h = Height above the spheroid





Testing and Validation

I tested the accuracy of this method by calculating the surface area within a large number of randomly-generated polygons scattered over central Arizona and eastern New Mexico, and comparing the surface area values with values computed using Triangulated Irregular Networks (TINs). Details of these tests are available from the author or from Calculating landscape surface area from digital elevation models (2004; Wildlife Society Bulletin, 32(3):829-839). In short, I found that this method produces highly accurate surface area measures within arbitrary polygon boundaries provided that a sufficiently large number of elevation cells were used in the analysis (for most purposes, n > 250 may be sufficient), with accuracy increasing as the proportion of cells completely contained within the polygon boundary increases.