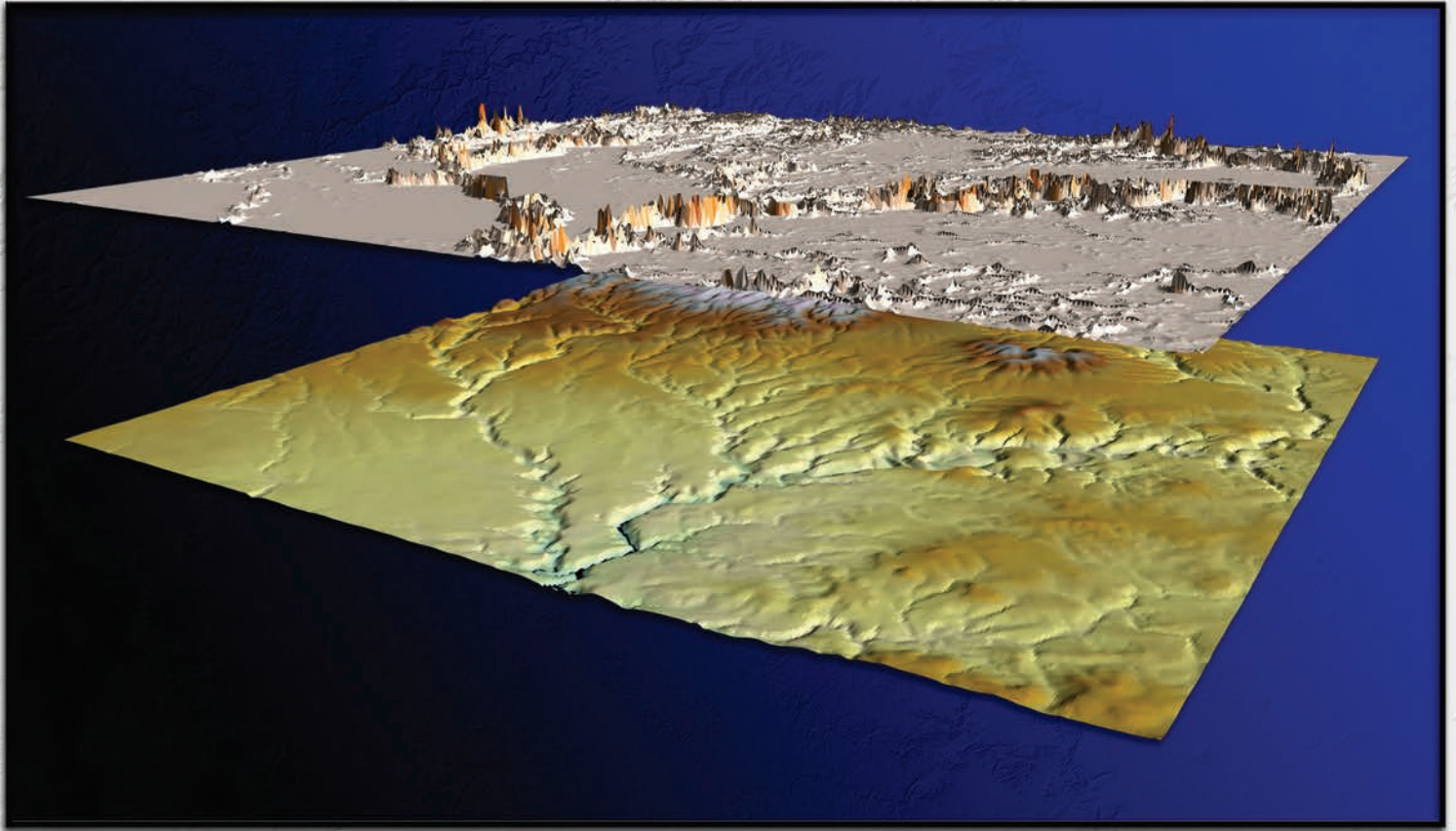
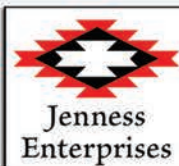
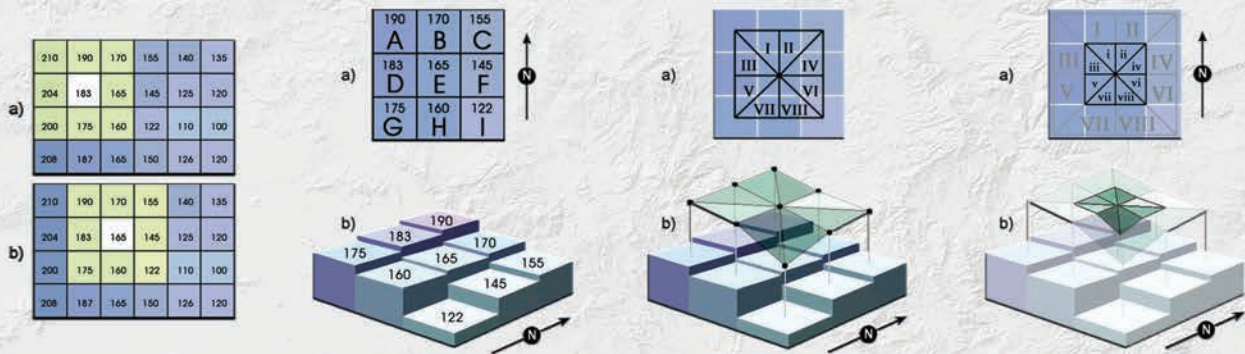


DEM Surface Tools



An ArcGIS Extension for Analyzing Raster Elevation Datasets



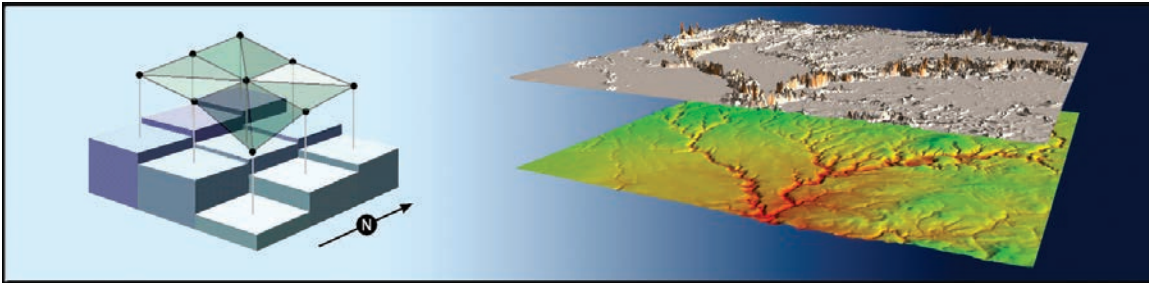
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About the Author

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NAME: DEM Surface Tools

Install File: surface_area.exe

Last modified: May 13, 2013

TOPICS: Sphere, geodesic, great circle, surface, area, ratio, elevation, topography, roughness, convolutedness, rugosity, terrain, geomorphology, slope, aspect, curvature, MDOW, hillshade, plan, profile

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Installing DEM Surface Tools

For ArcGIS 9.x

First close ArcGIS if it is open. The tools do not install properly if ArcGIS is running during the installation.

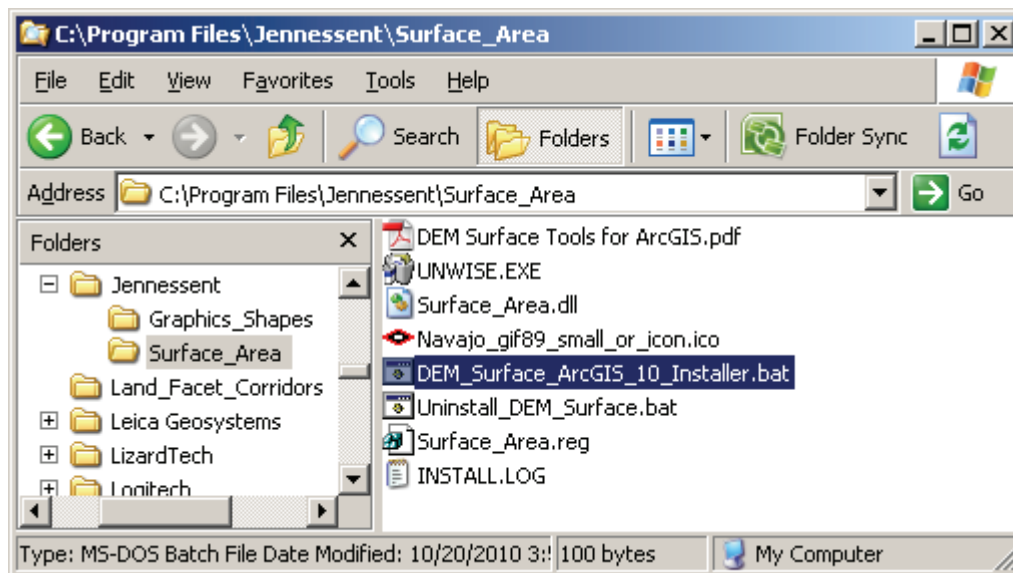
Install the DEM Surface Tools extension by double-clicking on the file **DEM_Surface_Tools.exe** (available at http://www.jennessent.com/downloads/DEM_Surface_Tools.zip, or from a link on the page http://www.jennessent.com/arcgis/surface_area.htm) and following the instructions. The installation routine will register the Surface_Area.dll with all the required ArcMap components.

The default install folder for the extension is named “Surface_Area” and is located inside the folder “Program Files\Jennessent”. This folder will also include some additional files and this manual.

For ArcGIS 10.0

Note: This function will only work if you have ArcGIS 10 installed.

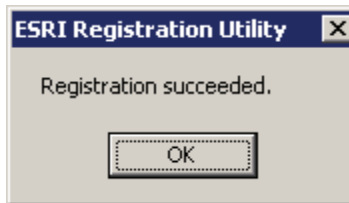
1. **First close ArcGIS if it is open.** The tools do not install properly if ArcGIS is running during the installation.
2. Install the DEM Surface Tools files by double-clicking on the file **DEM_Surface_Tools_10.exe** (available at http://www.jennessent.com/downloads/DEM_Surface_Tools_10.zip, or from a link on the page http://www.jennessent.com/arcgis/surface_area.htm) and following the instructions. This installation routine will install the Surface_Area.dll and several ancillary files on your hard drive, but will **not** register the tools with ArcGIS.
3. Use Windows Explorer to open your installation folder. If you used the default values, then this folder will be located at “Program Files\Jennessent\Surface_Area”. This folder will also include some additional files and this manual.



4. **For Windows XP:** Double-click the file “DEM_Surface_ArcGIS_10_Installer.bat” to register all the tools with ArcGIS 10.

For Windows Vista or Windows 7: Right-click the file “DEM_Surface_ArcGIS_10_Installer.bat” and click “Run as Administrator” to register all the tools with ArcGIS 10.

If the registration is successful, then you should see a "Registration Succeeded" notice.



Note: For the concerned or curious, the batch file *DEM_Surface_ArcGIS_10_Installer.bat* contains the following single line of text:

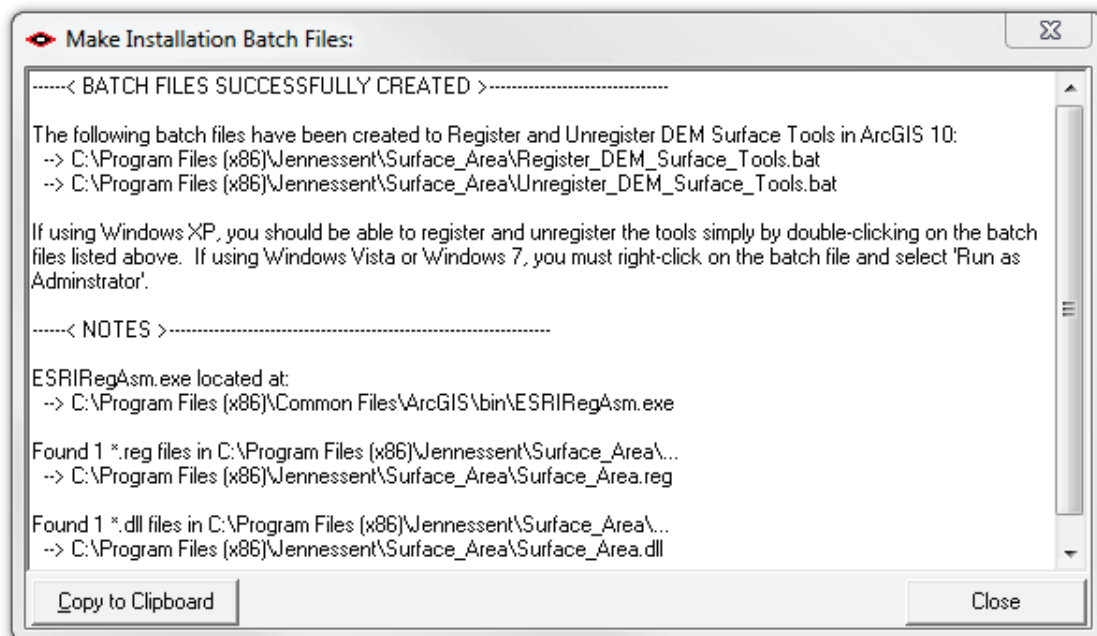
```
"%CommonProgramFiles%\ArcGIS\bin\ESRIRegAsm.exe" /p:Desktop "Surface_Area.dll"  
/f: "Surface_Area.reg"
```

It directs the ESRI installer *ESRIRegAsm* to register the DEM Surface Tools DLL *Surface_Area.dll* within ArcGIS, using GUID and Class ID values from the registry file *Surface_Area.reg* (also located in your installation directory). Both *DEM_Surface_ArcGIS_10_Installer.bat* and *Surface_Area.reg* may be opened and viewed using standard text editors such as Notepad or WordPad.

- 5. Alternative Method if you do not get the "Registration Succeeded" message:** If the method above does not work, the reason is probably due to the "%CommonProgramFiles" environmental variable pointing to the wrong location, and/or Windows Vista or Windows 7 Security settings. The fix is to use a batch file that includes the full pathnames to "ESRIRegAsm.exe" and to the extension DLL and REG files. You may edit the BAT file yourself, or you may use the tool **Make_Batch_Files.exe** (located in your installation folder) to create new registration and unregistration batch files that are properly formatted to your system.

If using Windows XP: Simply double-click on the file **Make_Batch_Files.exe** to create the new batch files.

If using Windows Vista or Windows 7: Right-click on the file **Make_Batch_Files.exe** and click "Run as Administrator" to create the new batch files.

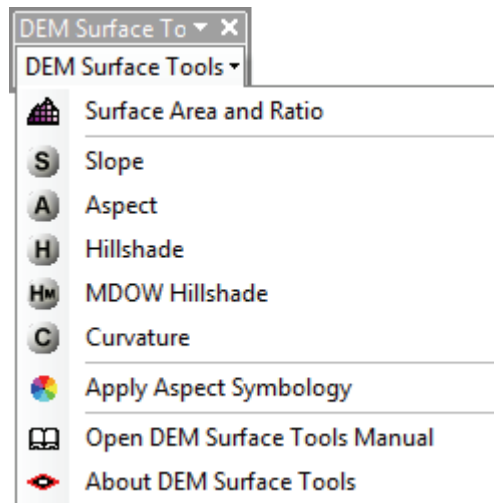


Repeat Step 4 above to register the tools in ArcGIS 10, but this time use the new BAT file *Register_DEM_Surface_Tools.bat*.

Viewing the Tools

These tools are installed as an extension in ArcMap, but it is a type of extension that is automatically loaded. You will **not** see this extension in the “Extensions” dialog available in the ArcGIS “Tools” menu. It is not dependent on any other extensions or any ArcGIS license level.

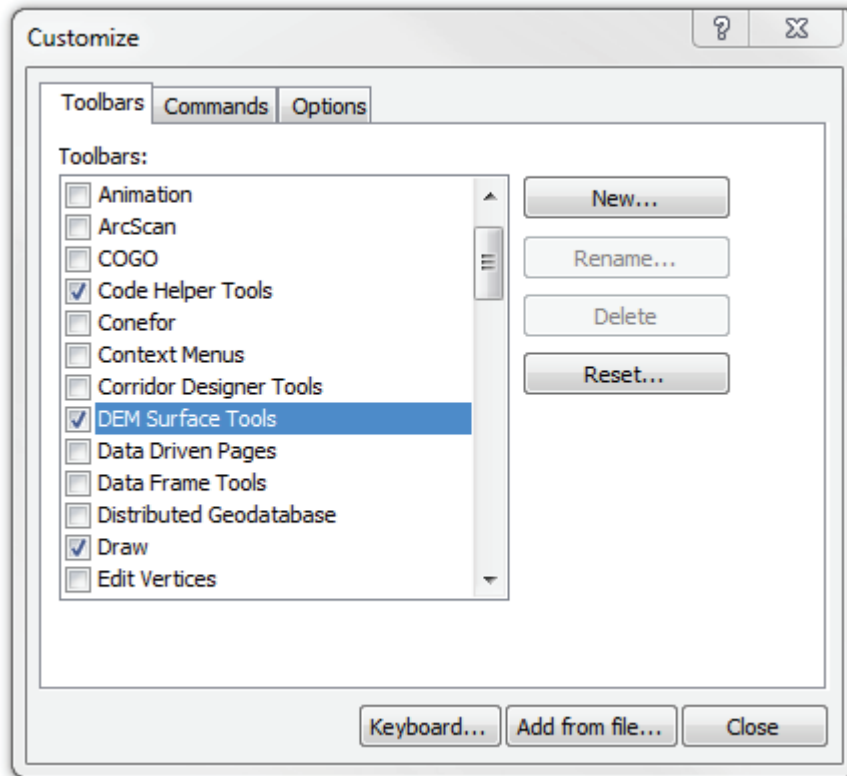
After installing the extension, you should see the following new toolbar in your map (it may also be embedded in your standard ArcMap toolbars, rather than as a standalone object):



If you do not see this toolbar, then open your “Customize” tool by either:

- 1) Double-clicking on a blank part of the ArcMap toolbar, or
- 2) Clicking the “Tools” menu, then “Customize”.

In the “Customize” dialog, click the “Toolbars” tab and check the box next to “DEM Surface Tools”:



You should now see the *DEM Surface Tools* toolbar.

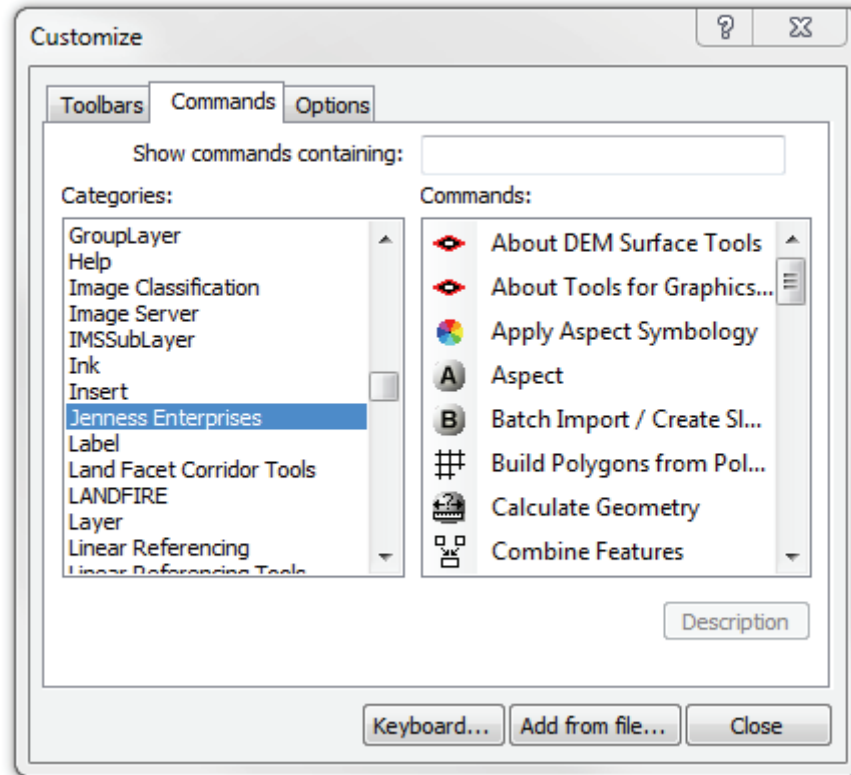
Copying and Adding the Tools to Other Toolbars

Because of the way ArcGIS handles toolbars and command buttons, you may add DEM Surface Tools command buttons to any toolbar you wish.

To do this, open your “Customize” tool by either:

- 1) Double-clicking on a blank part of the ArcMap toolbar, or
- 2) Clicking the “Tools” menu, then “Customize”.

In the “Customize” dialog, click the “Commands” tab and scroll down to select “Jenness Enterprises”:

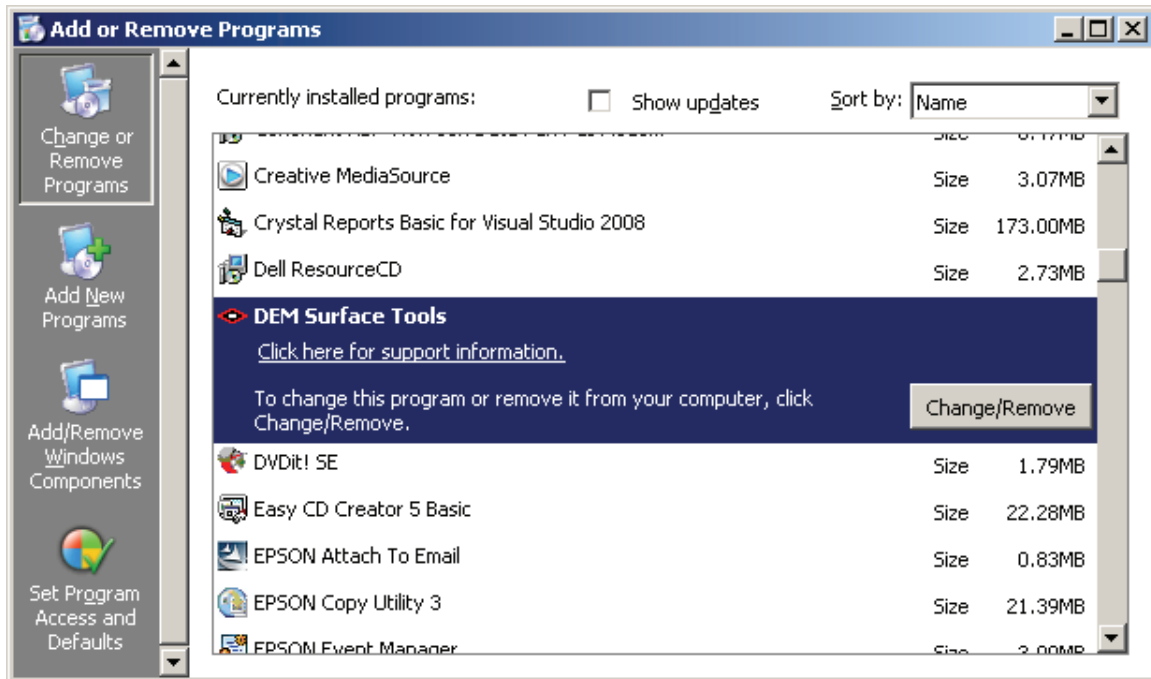


Finally, simply drag any of the commands out of the Customize dialog up into any of the existing ArcGIS toolbars.

Uninstalling DEM Surface Tools

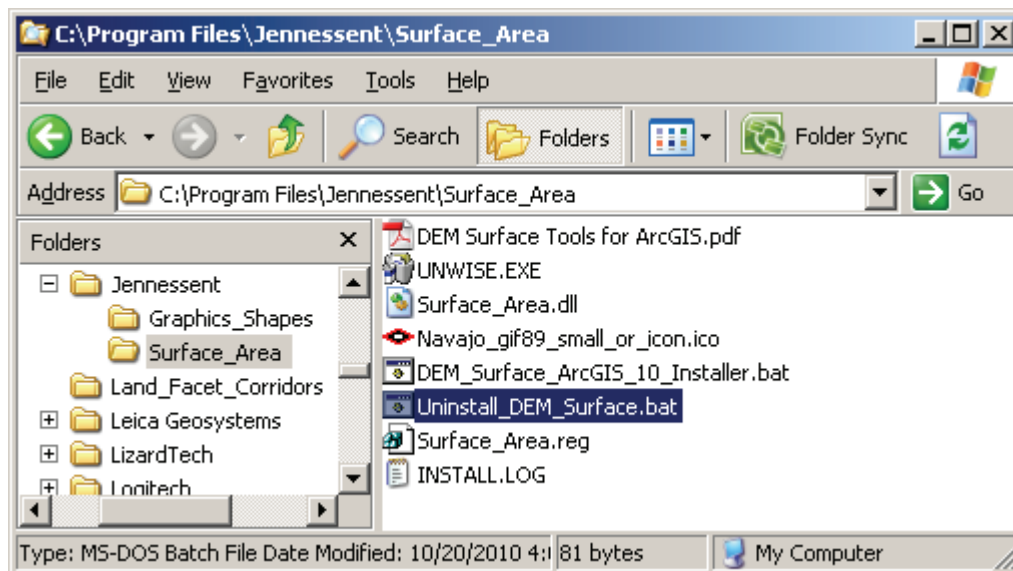
For ArcGIS 9.x

- 1) Close ArcGIS if it is open.
- 2) Click the Start button.
- 3) Open your Control Panel.
- 4) Double-click "Add or Remove Programs".
- 5) Scroll down to find and select "DEM Surface Tools."
- 6) Click the "Remove" button and follow the directions.



For ArcGIS 10.0

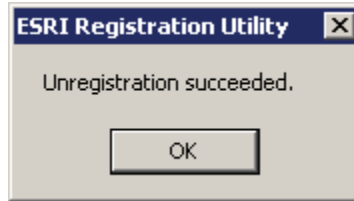
- 1) Close ArcGIS if it is open.
- 2) Use Windows Explorer to open your installation folder. If you used the default values, then this folder will be located at “Program Files\Jennessent\Surface_Area”. This folder will also include some additional files and this manual.



- 3) **For Windows XP:** Double-click the file “Uninstall_DEM_Surface.bat” to unregister all the tools.

For Windows Vista or Windows 7: Right-click the file “Uninstall_DEM_Surface.bat” and click “Run as Administrator to unregister all the tools.

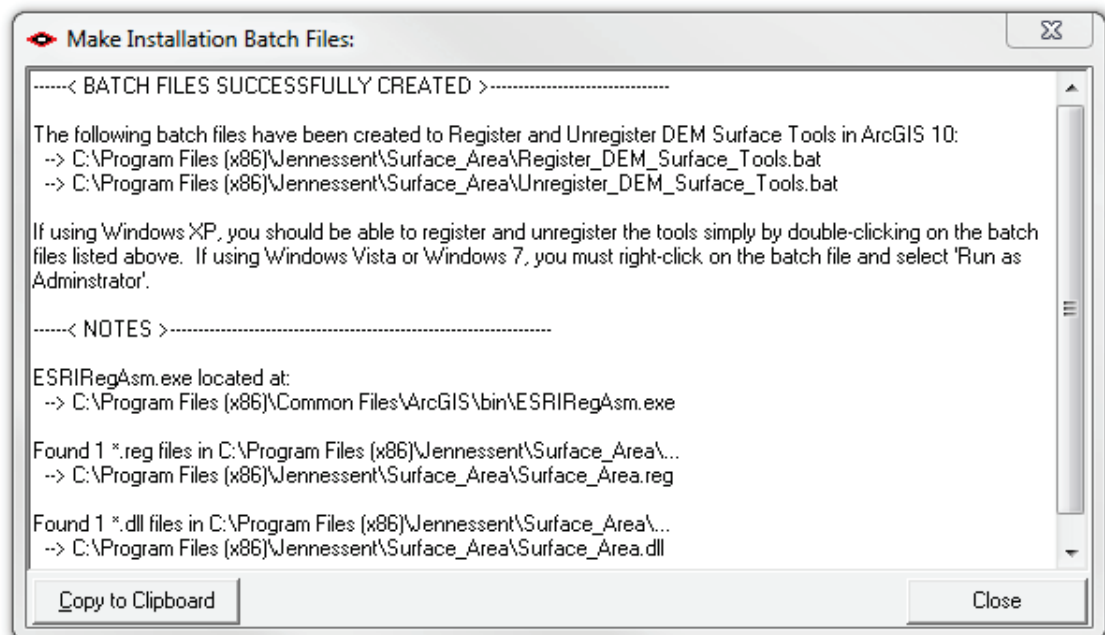
If the unregistration is successful, then you should see an “Unregistration Succeeded” notice.



- 4) **Alternative Method if you do not get the “Unregistration Succeeded” message:** If the method above does not work, the reason is probably due to the “%CommonProgramFiles” environmental variable pointing to the wrong location, and/or Windows Vista or Windows 7 Security settings. The fix is to use a batch file that includes the full pathnames to “ESRIRegAsm.exe” and to the extension DLL and REG files. You may edit the BAT file yourself, or you may use the tool **Make_Batch_Files.exe** (located in your installation folder) to create new registration and unregistration batch files that are properly formatted to your system.

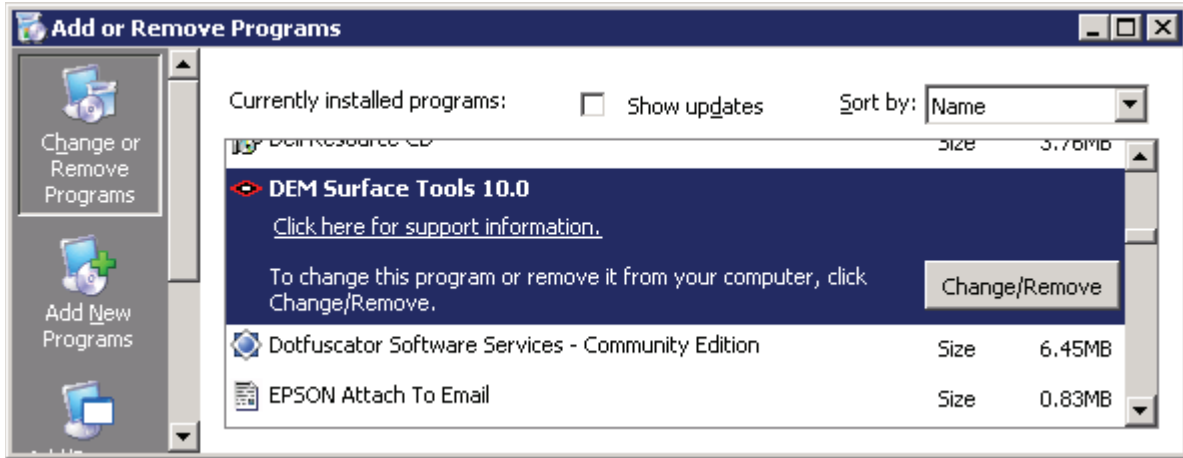
If using Windows XP: Simply double-click on the file **Make_Batch_Files.exe** to create the new batch files.

If using Windows Vista or Windows 7: Right-click on the file **Make_Batch_Files.exe** and click “Run as Administrator” to create the new batch files.



Repeat Step 3 above to unregister the tools in ArcGIS 10, but this time use the new BAT file *Unregister_DEM_Surface_Tools.bat*.

- 5) Click the Start button.
- 6) Open your Control Panel.
- 7) Double-click “Add or Remove Programs”.
- 8) Scroll down to find and select “DEM Surface Tools 10”.
- 9) Click the “Remove” button and follow the directions.



Note: For the concerned or curious, the batch file *Uninstall_DEM_Surface.bat* contains the following single line of text:

```
"%CommonProgramFiles%\ArcGIS\bin\ESRIRegAsm.exe" /p:Desktop /u "Surface_Area.dll"
```

It directs the ESRI installer *ESRIRegAsm* to unregister the DEM Surface Tools DLL *Surface_Area.dll* within ArcGIS. *Uninstall_DEM_Surface.bat* may be opened and viewed using standard text editors such as Notepad or WordPad.

Troubleshooting

If Any of the Tools Crash

If a tool crashes, you should see a dialog that tells us what script crashed and where it crashed. I would appreciate it if you could copy the text in that dialog, or simply take screenshots of the dialog and email them to me at jeffj@jennessent.com. **Note:** Please make sure that the line numbers are visible in the screenshots! The line numbers are located on the far right side of the text. Use the scrollbar at the bottom of the dialog to make the line numbers visible.

“Object variable or With block variable not set” Error:

If you open ArcMap and immediately see the error dialog appear with one or more error messages stating that “Object variable or With block variable not set”, then 90% of the time it is because ArcGIS was running when you installed the extension. The “Object” variable being referred to is the “Extension” object, and ArcGIS only sets that variable when it is initially turned on.

The solution is usually to simply close ArcGIS and restart it. If that does not work, then:

- 1) Close ArcGIS
- 2) Reinstall the extension
- 3) Turn ArcGIS back on.

RICHTX32.OCX Error (also comct332.ocx, comdlg32.ocx, mscmct2.ocx, mscmctl.ocx, msstdfmt.dll errors):

If you see a line in the error dialog stating:

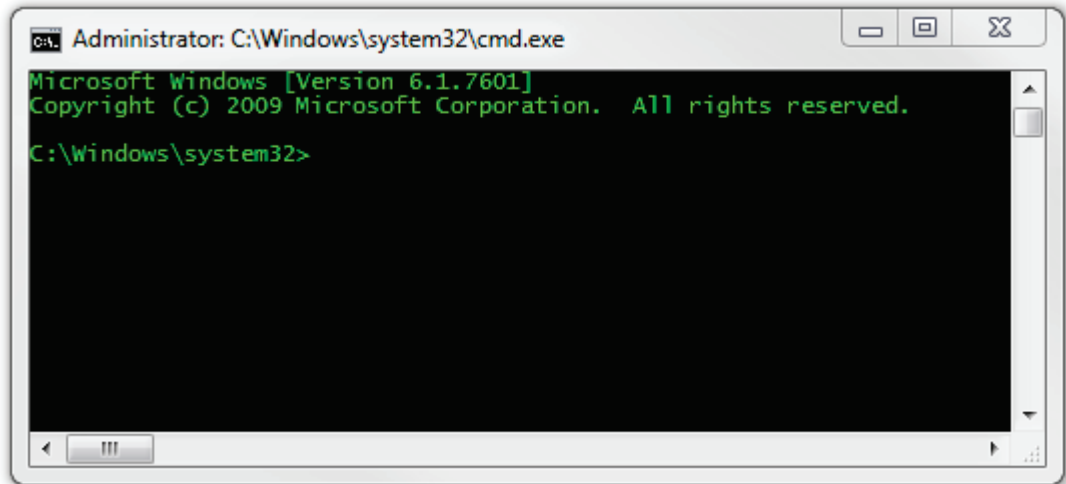
```
Component 'RICHTX32.OCX' or one of its dependencies not correctly registered: a file is missing or invalid
```

Or if you see a similar error stating that one or more of the files *comct332.ocx*, *comdlg32.ocx*, *mscmct2.ocx*, *mscmctl.ocx* or *msstdfmt.dll* are missing or invalid, then simply follow the instructions for *RICHTX32.OCX* below, but substitute the appropriate file for *RICHTX32.OCX*.

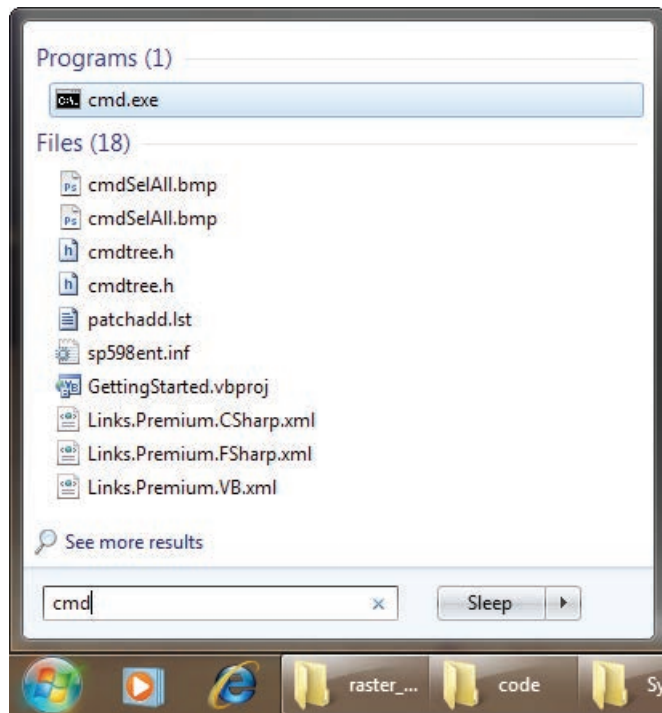
This error is almost always due to the fact that new installations of Windows 7 and Windows Vista do not include a file that the extension expects to find. For example, the file “richtx32.ocx” is actually the “Rich Text Box” control that appears on some of the extension dialogs. The other files refer to other common controls that might appear on the various extension dialogs.

The solution is to manually install the missing file (*richtx32.ocx*) yourself. Here is how to do it:

- 1) Open Windows Explorer and locate the file *richtx32.ocx* in your extension installation file.
- 2) If you are running a **32-bit version of Windows**, then copy *richtx32.ocx* to the directory
C:\Windows\System32\
If you are running a **64-bit version of Windows**, then copy *richtx32.ocx* to the directory
C:\Windows\SysWOW64\
- 3) Open an “Elevated Command Prompt” window. This is the standard Windows Command Prompt window, but with administrative privileges enabled. You need these privileges enabled in order to register the OCX with Windows. **Note:** The Elevated Command Prompt opens up in the “..\windows\system32” directory, not the “..\Users\[User Name]” directory. The window title will also begin with the word “Administrator:”



- a. **Method 1:** Click the "Start" button, then "All Programs", then "Accessories" and then **right-click** on "Command Prompt" and select **Run as Administrator**.
- b. **Method 2:** Click the "Start" button, and then click on the "Search Programs and Files" box. Type "cmd" and then click CONTROL+SHIFT+ENTER to open the Command window with Administrator privileges.



For more help on opening an Elevated Command Prompt, please refer to:

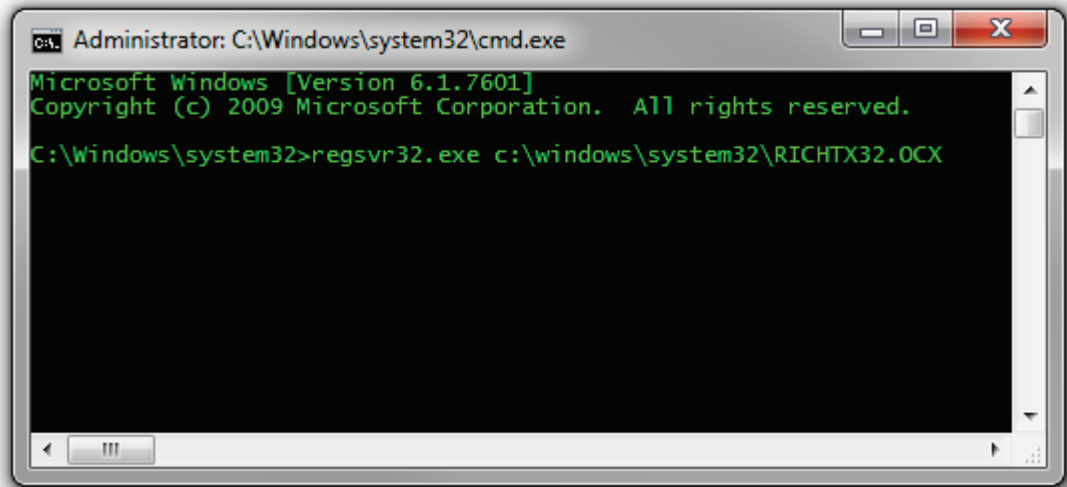
<http://www.sevenforums.com/tutorials/783-elevated-command-prompt.html>

<http://www.winhelponline.com/articles/158/1/How-to-open-an-elevated-Command-Prompt-in-Windows-Vista.html>

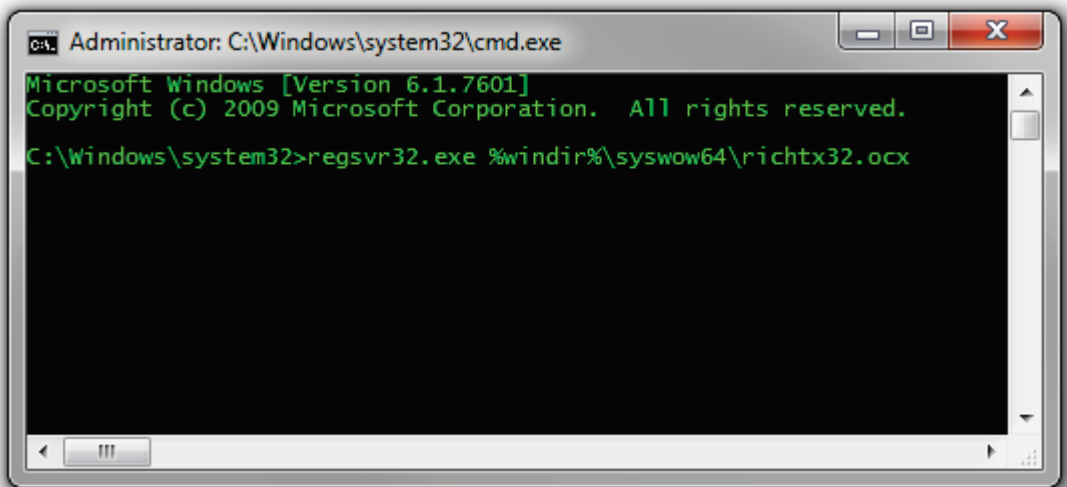
Or simply do a search for "Elevated Command Prompt".

- 4) Register the file *richtx32.ocx* using the Windows RegSvr function:

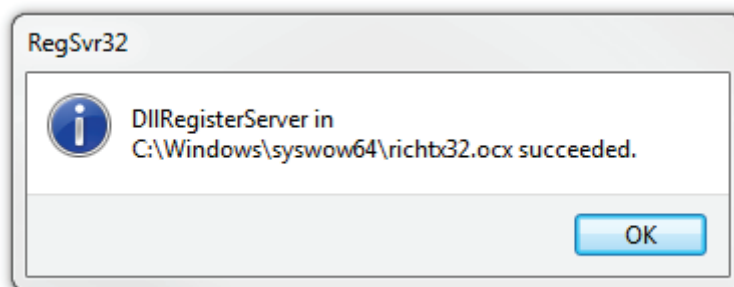
- a. If using a **32-bit version of Windows**, type the line
`regsvr32.exe c:\windows\system32\richtx32.ocx`



- b. If using a **64-bit version of Windows**, type the line
`regsvr32.exe %windir%\syswow64\richtx32.ocx`



- c. Click [ENTER] and you should see a message that the registration succeeded.



Introduction:

This extension provides you with tools to generate a variety of surface characteristics of a landscape, using both projected and unprojected (i.e. latitude / longitude) digital elevation model (DEM) rasters. These tools include:

- 1) Surface Area and Ratio of a landscape
- 2) Slope, with 3 slope algorithm options
- 3) Aspect, with 3 aspect algorithm options
 - This extension includes a tool to apply a traditional 9-category symbolization scheme to an existing Aspect raster layer (*Flat, North, Northwest, West, Southwest, South, Southeast, East, and Northeast*). This tool is useful if you load an aspect raster into ArcMap and wish to classify it by direction.
- 4) Hillshade, with optional hypsometric tinting
- 5) Multi-directional Oblique-Weighted Hillshade, with optional hypsometric tinting
- 6) Seven types of Landscape Curvature

ArcGIS has many useful tools for calculating slope, aspect, hillshade and curvature from DEM rasters. This extension offers some advantages over these existing tools in that:

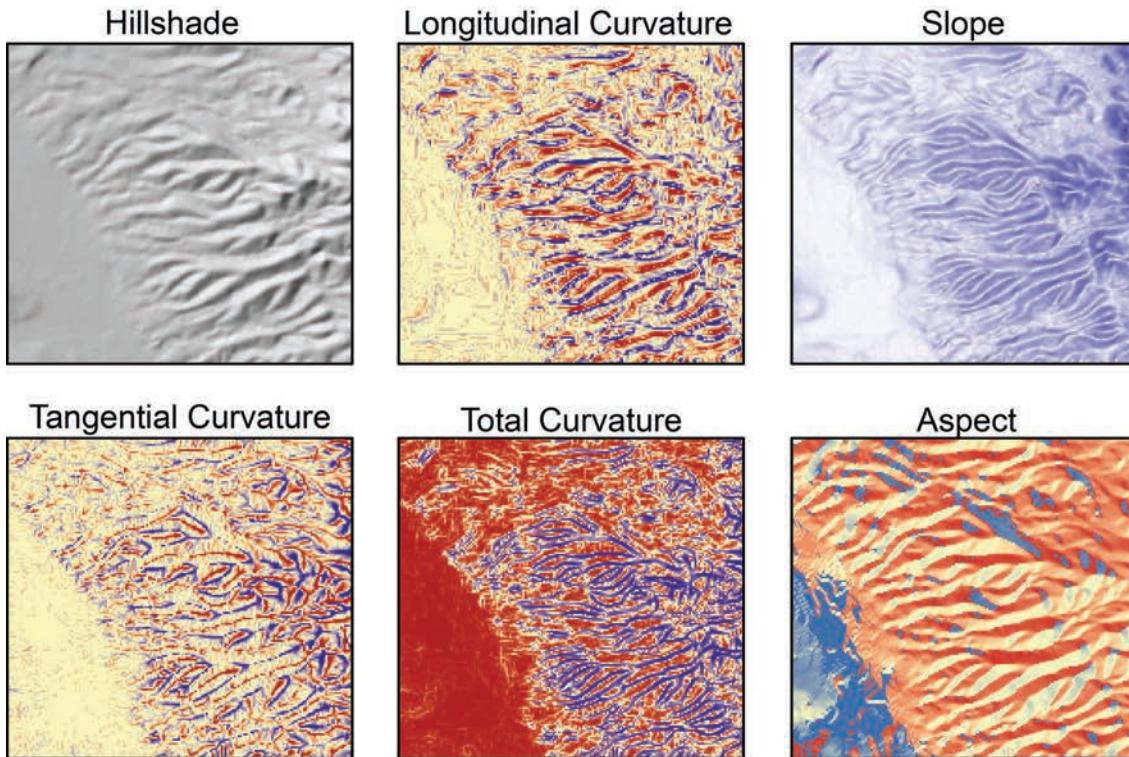
- 1) **These tools correctly handle Latitude / Longitude Data:** The existing ArcGIS tools typically do not work well with data in a geographic coordinate system (i.e. Latitude / Longitude coordinates). They expect the vertical units of a digital elevation model to be the same as the horizontal units, and at best they only allow you to enter in an adjustment factor (generally called a “Z-factor”) to adjust your Z-units to your X/Y-units. This Z-factor adjustment method is cumbersome to implement with latitude/longitude coordinate systems because the correct Z-factor is both hard to calculate and changes with latitude. Geographic data is especially confounding because the X-units are not the same as the Y-units. Both are in “degrees”, but a degree in latitude (Y-units) is not the same distance as a degree in longitude (X-units).

Unfortunately, most free elevation data available online is only available in latitude/longitude coordinates. This data can be projected, but projecting raster data introduces errors in resolution and precision (see horizontal and vertical artifacts on illustration on next page). This extension modifies the algorithms used to calculate common morphological characteristics of the landscape so that they accurately work with latitude/longitude data without projecting the data or using Z-factor adjustments.

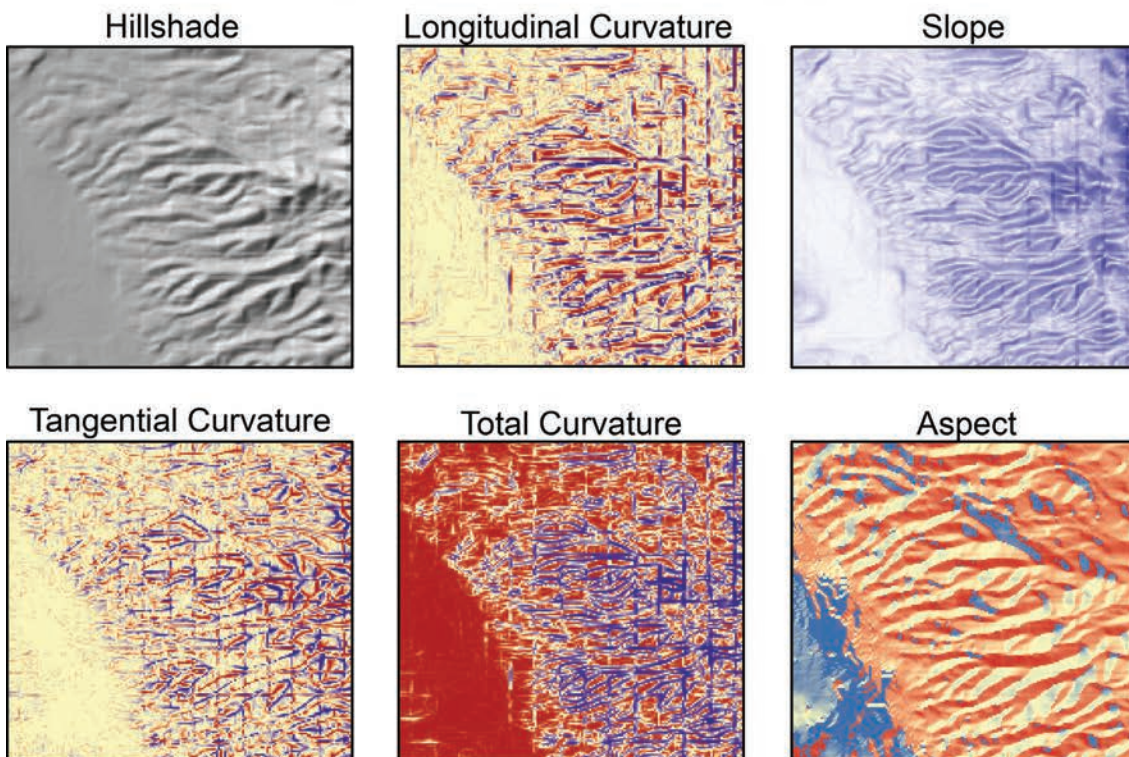
All methods are described in detail in the manual, but in general this extension will always automatically detect the coordinate system of the raster and use the method appropriate for that coordinate system.

- 2) **These tools offer several options not available in the standard ArcGIS tools:** Including multiple slope and aspect algorithms, optional hypsometric shading for hillshades, the MDOW model for hillshades, and many more options for landscape curvature.
- 3) **Surface Area and Ratio:** This extension allows you to generate Surface Area and Surface Ratio rasters from an existing Elevation raster. The cell values for these new rasters reflect the surface area and (surface area) / (planimetric area) ratio for the land area contained within that cell's boundaries. Surface Ratio provides a useful index of topographic roughness and convolutedness, while Surface Area give a more realistic estimate of the land area available than you can get from the simple planimetric area.
- 4) **These tools do not require Spatial Analyst:** This extension uses only standard ArcGIS ArcObjects functions and therefore does not require a license for either Spatial Analyst or 3D Analyst.

Calculated directly from Latitude / Longitude DEM

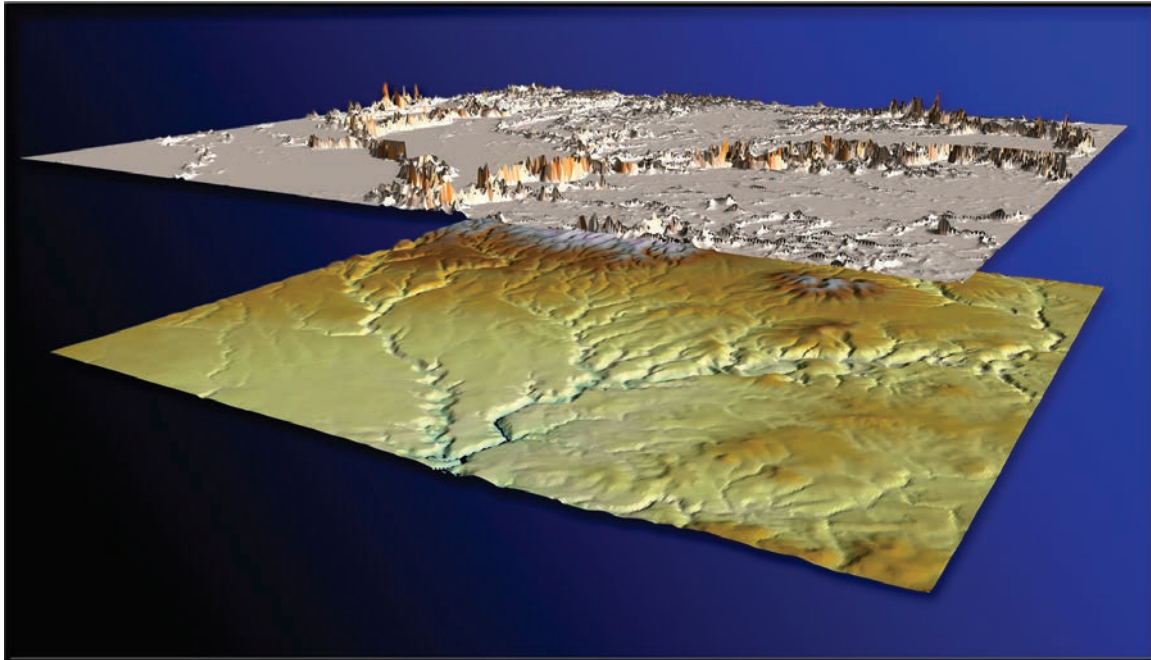


Calculated from Projected DEM



DEM Surface Tools

Surface Area and Ratio:



BACKGROUND

Landscape area is almost always presented in terms of planimetric area, as if a square kilometer in the Himalayas represents the same amount of land area as a square kilometer in the Nebraska. Predicted home ranges for wildlife species generally use planimetric area even when describing mountain-dwelling species such as mountain goats (*Oreamnos americanus*) and puma (*Felis concolor*). But if a species' behavior and population dynamics are a function of available resources, and if those resources are spatially limited, then these resources might be better assessed using surface area of the landscape.

Surface area also is a basis for a useful measure of landscape topographic roughness. The surface area ratio of any particular region on the landscape can be calculated by dividing the surface area of that region by the planimetric area. For example, Bowden et al. (2003) found that ratio estimators of Mexican spotted owl (*Strix occidentalis lucida*) population size were more precise using a version of this surface area ratio than with planimetric area.

There are a variety of methods in the literature for measuring terrain irregularity. Hobson (1972) described some early computational methods for estimating surface area and discussed the concept of surface area ratios. Beasom (1983) described a method for estimating land surface ruggedness based on the intersections of sample points and contour lines on a contour map, and Jenness (2000) described a similar method based on measuring the density of contour lines in an area. Mandelbrot (1983:29, 112–115) described the concept of a “fractal dimension” in which the dimension of an irregular surface lies between 2 (representing a flat plain) and 3 (representing a surface that goes through every point within a volume). Calculating this fractal dimension can be very challenging computationally, and Polidori et al. (1991), Lam and De Cola (1993) and Lorimer et al. (1994) discussed a variety of methods for estimating the fractal dimension for a landscape.

An estimate of surface area also could be derived from slope and aspect within a cell (Berry 2002), although Hodgson (1995) demonstrated how most slope-aspect algorithms generate values reflecting an area 1.6–2 times the size of the actual cell. Surface area values derived with this method would, therefore, be unduly influenced by adjacent cells. Another problem with this method is that slope algorithms do not actually use the elevation of the central cell in the formula

(see p . 42), meaning that any additional surface area created by an especially high or low central cell is ignored. For those who are interested, however, the method is quick, easy and intuitive. Berry's method can be thought of as simply using the slope to calculate an adjustment factor for the cell planimetric area. If the slope is flat, then the adjustment factor is equal to 1 and therefore the surface area is equal to the planimetric area. As the slope increases, the adjustment factor is equal to:

$$\text{Adjustment Factor} = \frac{1}{\text{Cos}(\text{Slope})}$$

where Slope is in *Radians*, calculated as:

$$\text{Slope (Radians)} = \frac{[\text{Slope (Degrees)}]\pi}{180}$$

Therefore, assuming cell size is defined as the length along one edge of the cell, and Slope is measured in degrees, then surface area can be calculated as:

$$\text{Surface Area} = \frac{c^2}{\text{Cos}\left(S\left(\frac{\pi}{180}\right)\right)}$$

where:

c = Cell Size

S = Slope in Degrees


One important factor to note about this method is that, because the cell size is constant for all cells, the final surface area raster is perfectly correlated with the Slope raster. Therefore you wouldn't want to include both Slope and Surface Area in the same statistical model.

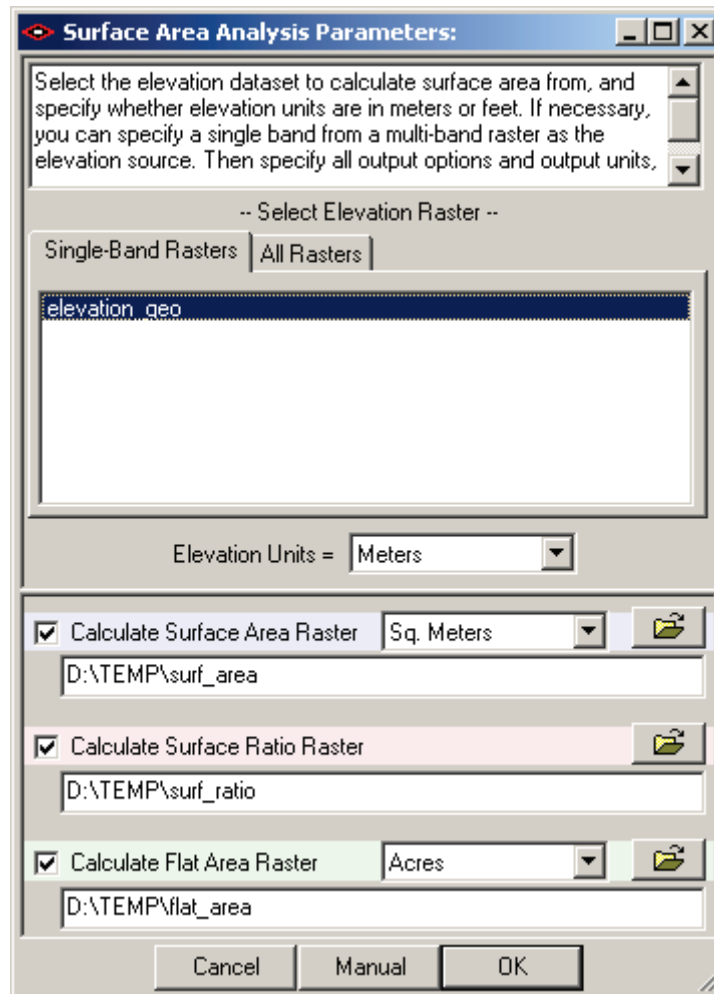
The methods used by this tool are described in the section following "Running the Surface Area Tool" below:

Note: This extension also calculates surface areas much faster than 3D Analyst, and with as much accuracy. However, because the output is in Raster format rather than Vector, surface areas within polygons cannot be determined as precisely as you can do with a TIN. Surface areas within a polygon are calculated as the sum of the surface area cell values of all those cells whose cell centers lie within the polygon boundaries.

Generating surface area in raster format offers the advantage of allowing you to conduct neighborhood analyses on Surface Area rasters, such that you can generate a raster of statistics describing all cells within a particular range of each cell in the raster. This is useful if you want to conduct analyses at different spatial scales.

RUNNING THE SURFACE AREA TOOL:

- 1) Load your elevation raster into ArcGIS.
- 2) Click the *Surface Area and Ratio* Button  to open the “Surface Area Analysis Parameters:” dialog.

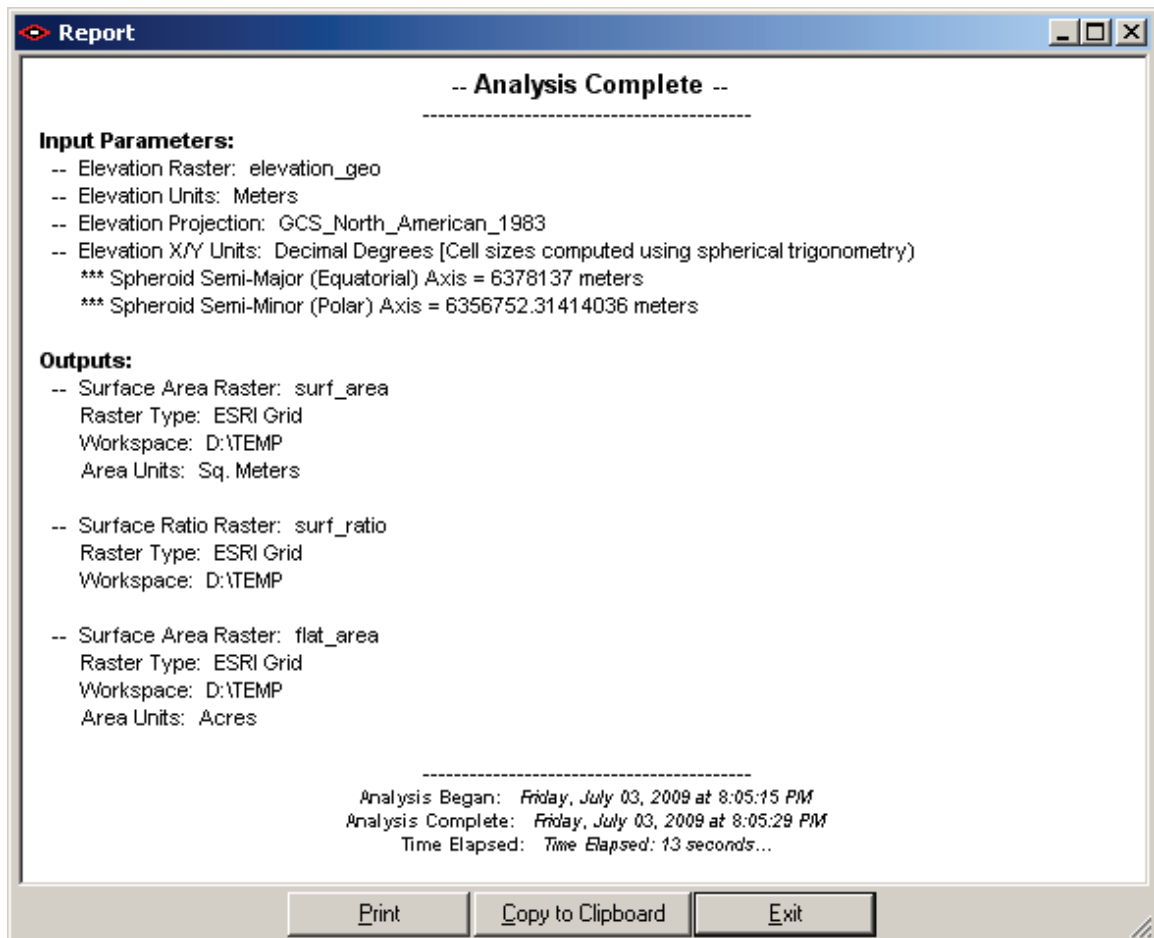


- 3) Select your elevation raster from the listbox of rasters in your map. If you wish to analyze a particular band of a multi-band raster, then click the “All Rasters” tab and select the specific band you wish to analyze.
- 4) Specify the elevation units of your dataset. The tool is currently written to only accept elevation values in meters or feet because those are by far the most common. Please contact the author (jeffj@jennessent.com) if you need additional elevation unit options.
- 5) Choose which of three output rasters you wish to generate. You may generate any or all of these in a single analysis.
 - a. *Surface Area*: This will produce a raster in which the cell values reflect the topographic surface area within that cell, calculated according to methods described later in this manual.

Note: You have the option to set the areal units for your output. Your options are *Square Meters, Hectares, Square Kilometers, Square Feet, Acres* and *Square Miles*.

All of these options are available regardless of whether your original elevation units are meters or feet, or whether your original coordinate system was metric, standard or geographic. The tool will make the appropriate conversion regardless of the original data units.

- b. *Surface Ratio*: This will produce a raster in which the surface area values of each cell are divided by the planimetric area of that cell. These values can potentially range from 1 to infinity, but in practice rarely reach as high as 3.
 - c. *Flat Area*: This is the planimetric area of the cell. If your raster is projected then all cells will have the same value, equal to (cell size)². If your raster is in geographic coordinates (i.e. latitude / longitude), then the cell sizes will increase as you get closer to the equator.
- Note:** You have the same options for areal units here as you do with the *Surface Area* raster.
- 6) Click "OK" to run the analysis. Upon completion, your rasters will be automatically added to your map and you will be shown a report detailing what the tool did.



SURFACE AREA AND RATIO METHODS

Methods used by this extension are described in:

[Jenness, J. S. 2004.](#) Calculating landscape surface area from digital elevation models. *Wildlife Society Bulletin*. 32(3):829-839

In summary, this extension calculates surface areas using the following strategy:

- For each cell in the raster, surface areas are based on triangle areas derived from eight triangles
- Each triangle connects the center point of the central cell with the center points of two adjacent cells. These triangles are located in three-dimensional space, so that the area of the triangle represents the true surface area of the space bounded by the three points.
- The triangle area is adjusted so that it only represents the portion of the triangle that overlays the central cell.
- The areas of the eight triangles are summed to produce the total surface area of that cell.
- The surface ratio of the cell is calculated by dividing the surface area of the cell with the planimetric area of the cell.

Triangulating the Surface

This extension derives surface areas for a cell by using information from the eight cells adjacent to the center cell. For example, given a sample elevation raster:

210	190	170	155	140	135
204	183	165	145	125	120
200	175	160	122	110	100
208	187	165	150	126	120

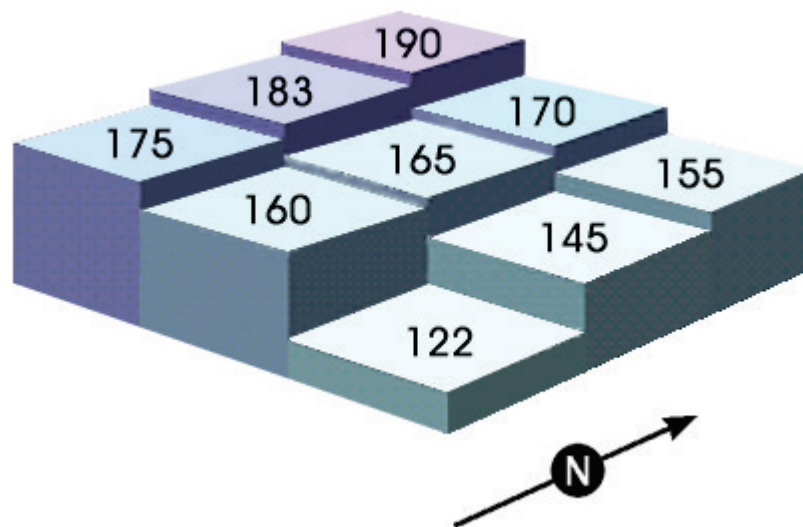
this extension would calculate the surface area for the cell with elevation value "183" based on the elevation values of the eight cells surrounding it,

210	190	170	155	140	135
204	183	165	145	125	120
200	175	160	122	110	100
208	187	165	150	126	120

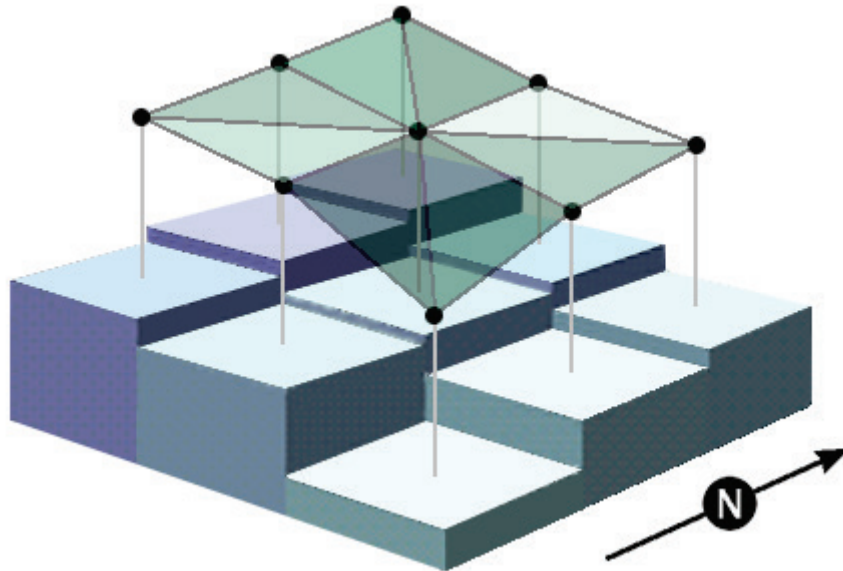
and the surface area for the "165" cell based on the eight elevation value surrounding it:

210	190	170	155	140	135
204	183	165	145	125	120
200	175	160	122	110	100
208	187	165	150	126	120

The following example of a Surface Area calculation will be based on the illustration above, where we will calculate the surface area for a cell with an elevation value of 165, surrounded by cells at 190, 170, 155, 183, 145, 175, 160 and 122. This raster of elevation values can be pictured in 3-dimensional space as a set of adjacent columns, each rising as high as its specified elevation value.



We take the 3-dimensional centerpoints of each of these 9 cells, and calculate the lengths of the 8 lines that connect the central cell's centerpoint with the center points of the 8 surrounding cells. We then calculate the lengths of the lines that connect each of the 8 surrounding cells with the one right next to it, so that we end up with the lengths of the sides of the 8 triangles that all meet at the center point of the central cell. Using these lengths, we can calculate the areas of each of the triangles.



These lengths are simple to calculate using the Pythagorean Theorem, which states that, in a right triangle, the square of the length of the hypotenuse is equal to the sum of the squares of the other two sides. Thus, for any two points **A** and **B**:

Pythagorean Theorem:

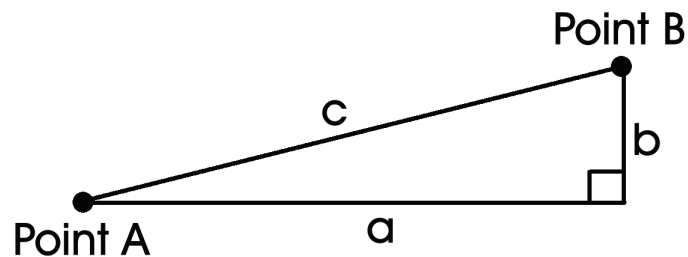
$$a^2 + b^2 = c^2 \quad \text{or} \quad c = \sqrt{a^2 + b^2}$$

where:

a = planimetric distance from Point A to Point B

b = difference in elevation between Point A and Point B

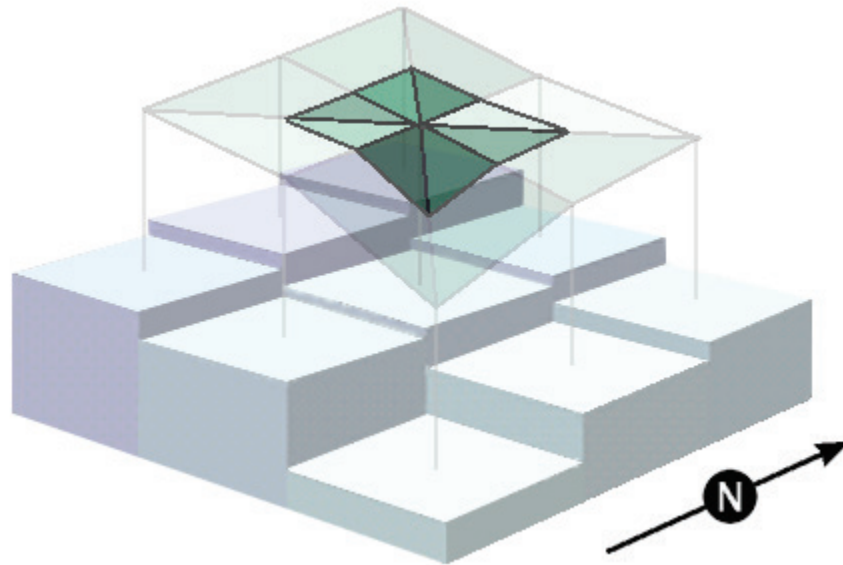
c = True surface distance from Point A to Point B



"**b**" is easy to calculate, since it is simply the difference between the two cell values. "**a**" is easy to get for the cells directly to the North, East, South and West, since it is simply the value of the cell size. For the diagonals, we use the Pythagorean theorem again and calculate

$$a = \sqrt{2(\text{cell size})^2}.$$

However, since we're only concerned with the surface area within each cell, we cut all the triangle lengths in half. Now when we calculate the area of a triangle based on the lengths of three sides, the three sides will represent the portion of the triangle that lies within the cell boundaries.



This action is justified based on the Side-Angle-Side similarity criterion for similar triangles (Euclid 1956:204), which states that “If two triangles have one angle equal to one angle and the sides about the equal angles proportional, the triangles will be equiangular and will have those angles equal which the corresponding sides subtend”. Each original triangle is “similar” to its corresponding clipped triangle because the 2 sides extending from the center cell in the original triangle are exactly twice as long as the respective sides in the clipped triangle, and the angles defined by these 2 sides are the same in each triangle. Therefore, the third side of the clipped triangle must be exactly half as long as the corresponding side of the original triangle.

Technical Note: The above description only describes the logic used to derive the surface area. The extension is not actually connecting any lines or even using any vector data at all. Rather, the extension examines each cell individually, generating the eight triangle areas in memory based on the 8 cells surrounding it. Finally, the extension adds those triangle areas to produce the final surface area value for that cell.

Calculating the Area of a Triangle from the Lengths of the Three Sides

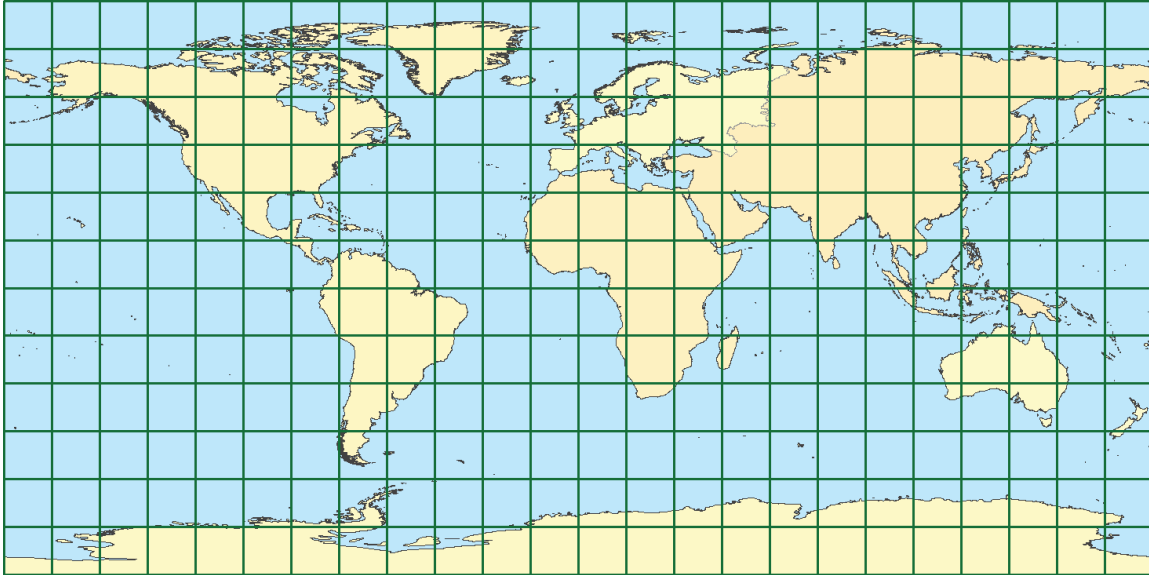
This extension uses the Triangle half-perimeter formula to calculate triangle areas, attributed to Heron of Alexandria and Archimedes (see also Abramowitz and Stegun [1972, p. 79]):

$$\text{Triangle half-perimeter} = S = \frac{a + b + c}{2}$$
$$\text{Area} = \sqrt{S(S - a)(S - b)(S - c)}$$

Modification for Geographic Coordinates

Latitude and longitude coordinate values present a special challenge to these calculations. Latitude/Longitude values are polar coordinates (i.e. they reflect angular units) rather than projected Cartesian coordinates (i.e. X,Y units), and the methods used to calculate area from Cartesian coordinates are not appropriate for polar coordinates. A triangle area reported in “square degrees” is completely nonsensical.

Furthermore, even though geographic raster cells may look square in a geographically projected map window, they most definitely are not square as they lay on the surface of the earth. For example, given a very coarse raster covering the entire planet with 15° cell sizes:



A change in 15° latitude represents approximately the same distance anywhere on the surface of the planet. However, the distance represented by 15° longitude depends on your latitude. On the equator, 15° longitude covers the same distance as 15° latitude. At the poles, 15° longitude covers no distance at all. The squares in the map above get progressively narrower as you approach the poles, and they are actually triangles at the poles themselves.



We can still use our basic methods described above, but we cannot use the original latitudes and longitudes of the cell centers. We can, however, convert these coordinates to 3-dimensional Cartesian coordinates.

Converting between Latitude / Longitude and Cartesian Coordinates

Note: The following section is adapted from the manual for the author's *Tools for Graphics and Shapes* extension, available at http://www.jennessent.com/arcgis/shapes_graphics.htm.

This tool calculates distances between cell centerpoints as they lay on the spheroid, rather than as they lay on the sphere. The shape of the earth is modeled better with a spheroid than a sphere so the calculations are slightly more accurate using equations designed for a spheroid.

However, the equations are easier to understand if they are introduced in their simplest form. Therefore this section begins by explaining the conversion methods on the sphere, and then shows how the equations are modified to handle the spheroid.

Latitude / Longitude to Cartesian Coordinate on the Sphere:

Given a Latitude and Longitude in Degrees, calculate X, Y and Z as follows:

$$X = R \sin \phi \cos \theta$$

$$Y = R \sin \phi \sin \theta$$

$$Z = R \cos \phi$$

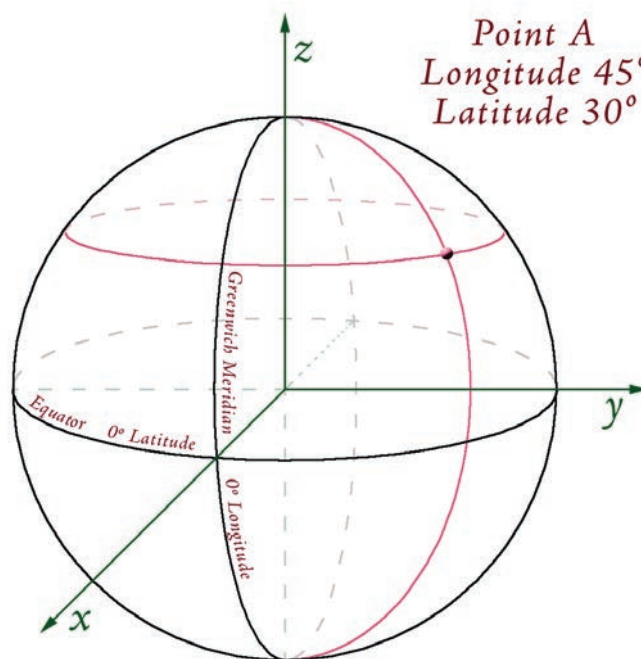
where:

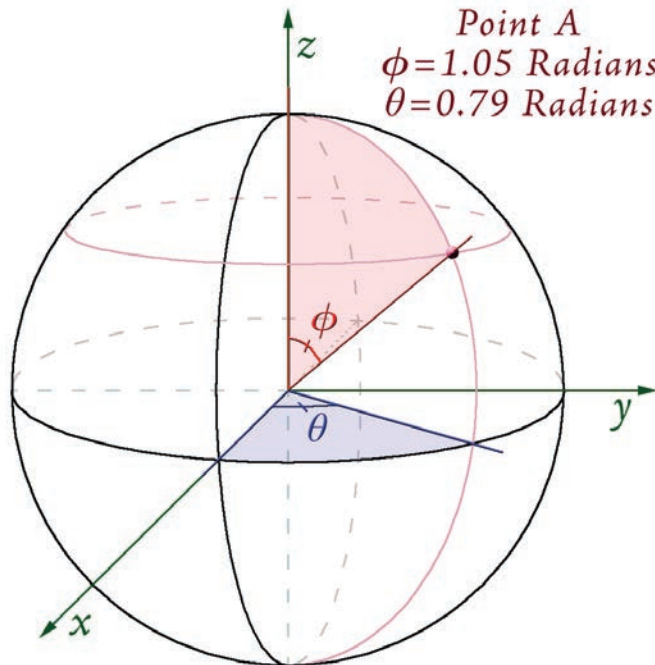
$$\phi = \frac{(90 - \text{Latitude})\pi}{180} = \text{Angle from North Pole down to Latitude, in Radians}$$

$$\theta = \frac{(\text{Longitude})\pi}{180} = \text{Angle from Greenwich to Longitude, in Radians}$$

R = Radius of Sphere

For example, a point at 30° Latitude and 45° Longitude would be converted to $\phi = 1.05$ radians and $\theta = 0.79$ radians:





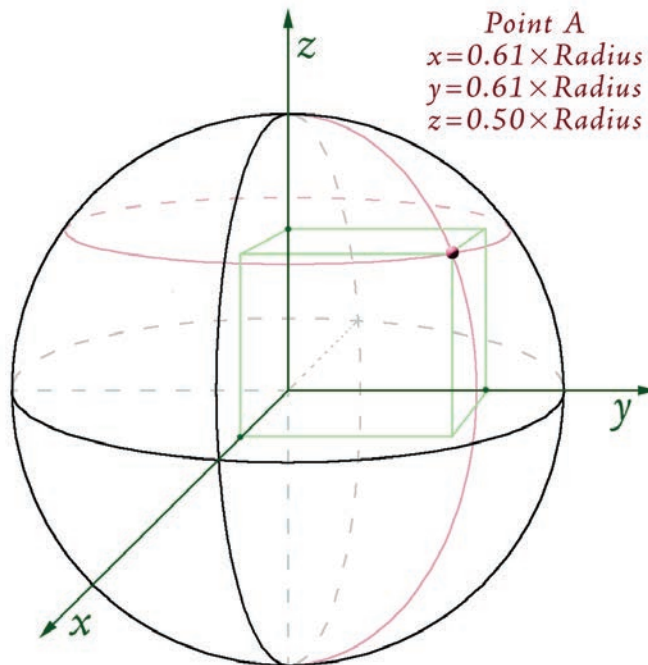
NOTE: Be careful with the terms ϕ and θ ! This document treats ϕ as the angle south from the North Pole, and θ as the angle east from Greenwich. However, the literature is not consistent with these two terms! Much of the literature uses these exact two symbols, but for the other angle (i.e. ϕ for east/west and θ for north/south).

Using the formula above, calculate X, Y and Z from ϕ and θ .

$$X = R \sin \phi \cos \theta = R \sin(1.05) \cos(0.79) = 0.61R$$

$$Y = R \sin \phi \sin \theta = R \sin(1.05) \sin(0.79) = 0.61R$$

$$Z = R \cos \phi = R \cos(0.79) = 0.5R$$



NOTE: As with ϕ and θ , the literature is inconsistent regarding how to write the X, Y and Z coordinates of a 3-dimensional point. If a point is located at X, Y, Z, some sources will write the coordinates as (X,Y,Z) while other sources will write it as (Z,X,Y). If you write coordinates in either of these formats, please identify which format you are using!

Latitude / Longitude to Cartesian Coordinates on the Spheroid:

As illustrated in Chapter 2 of Iliffe (2000), the formulas above can be adapted to oblate spheroids (such as a planet). If you refer to Iliffe's text, please be aware that his formulae define ϕ as Latitude, rather than as $(90 - \text{Latitude})$. The formulae below are modified from Iliffe's formulae to make them consistent with the conversion formulae for the sphere above.

Given a Latitude and Longitude in Degrees, Spheroid Equatorial and Polar Axes, and an optional height above the spheroid, calculate X, Y and Z as follows:

$$X = (v + h) \sin \phi \cos \theta$$
$$Y = (v + h) \sin \phi \sin \theta$$
$$Z = ((1 - e^2)v + h) \cos \phi$$

where:

$$\phi = \frac{(90 - \text{Latitude})\pi}{180} = \text{Angle from North Pole down to Latitude, in Radians}$$

$$\theta = \frac{(\text{Longitude})\pi}{180} = \text{Angle from Greenwich to Longitude, in Radians}$$

a = Spheroid equatorial radius (i.e. semi-major axis)

b = Spheroid polar radius (i.e. semi-minor axis)

$$e^2 = \text{Spheroid eccentricity (squared)} = \frac{a^2 - b^2}{a^2}$$

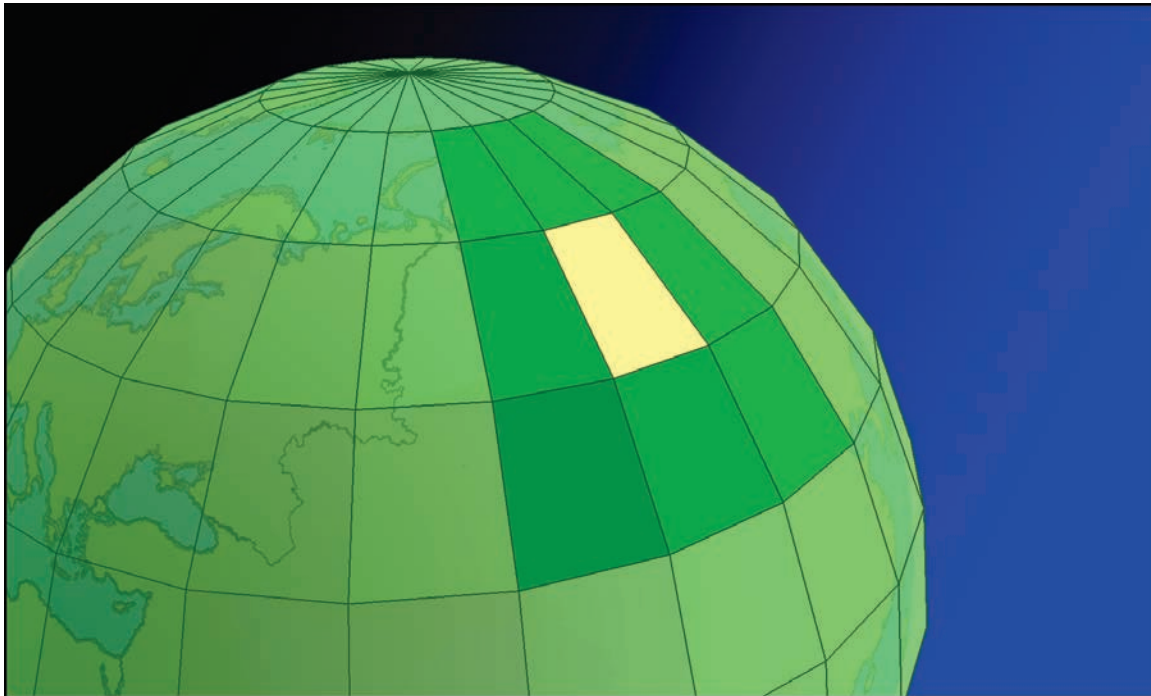
$$v = \frac{a}{\sqrt{1 - e^2 \cos^2 \phi}}$$

h = Height above the spheroid

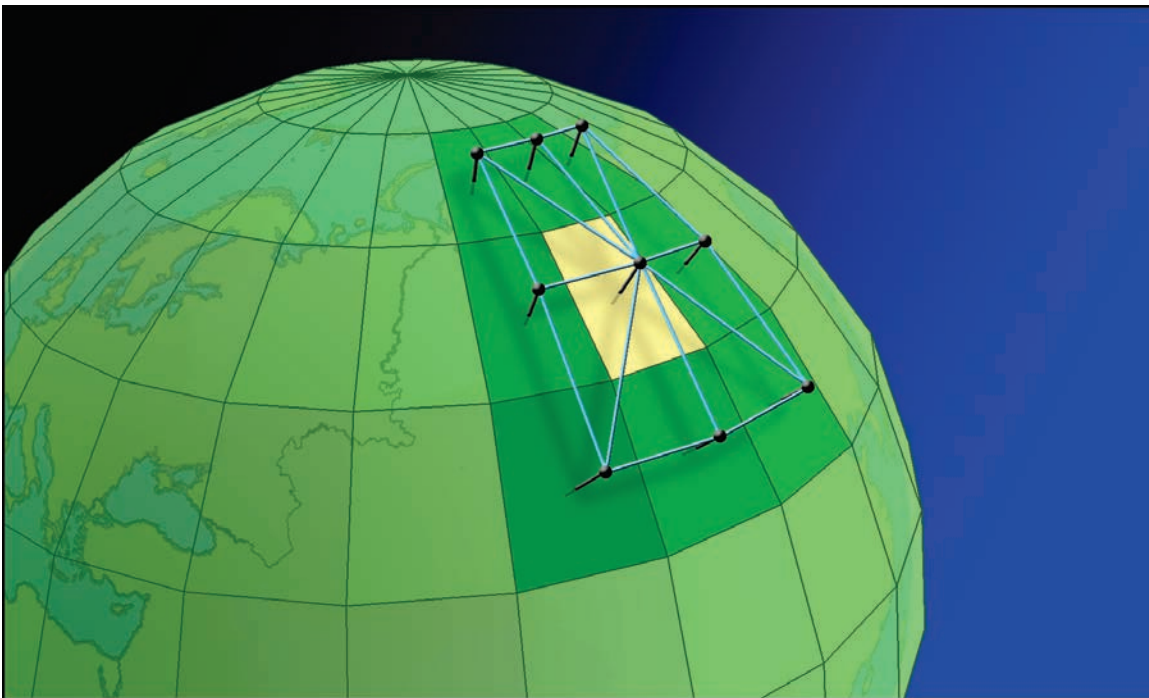
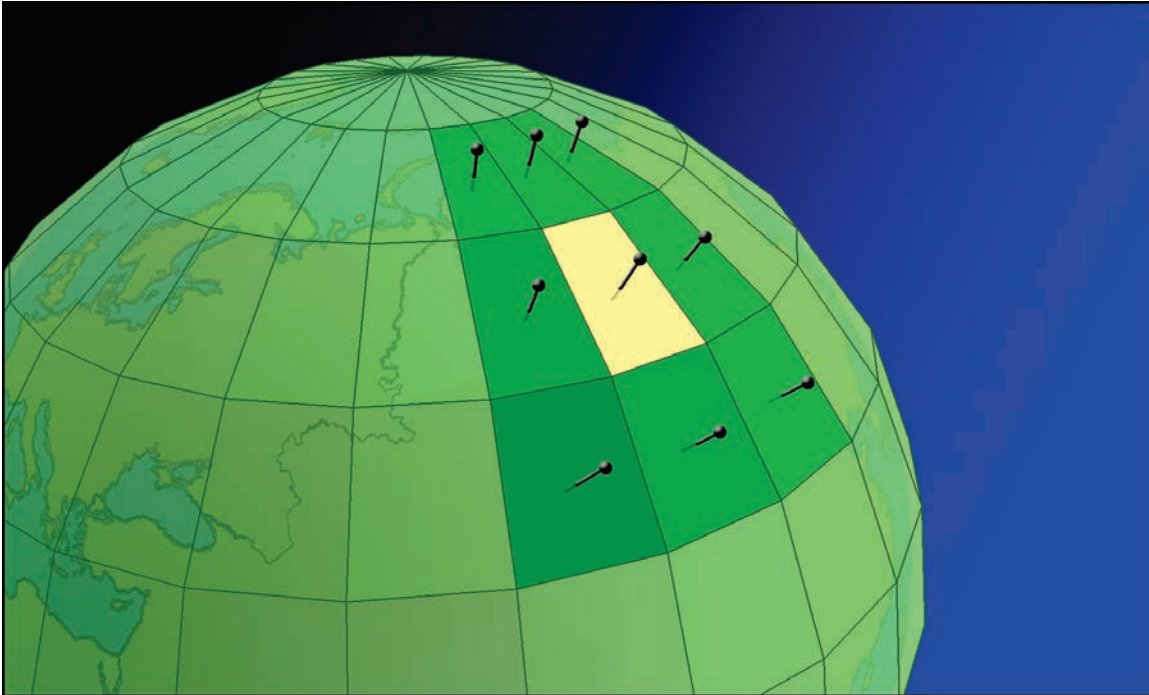
Creating Triangles on the Spheroid

Once the latitude and longitude coordinates are calculated, then the process is very similar to that for projected rasters. The main difference is that the cell centerpoints extend out from the curved surface of a planet rather than from a projected plane, but this makes no difference to the mathematics.

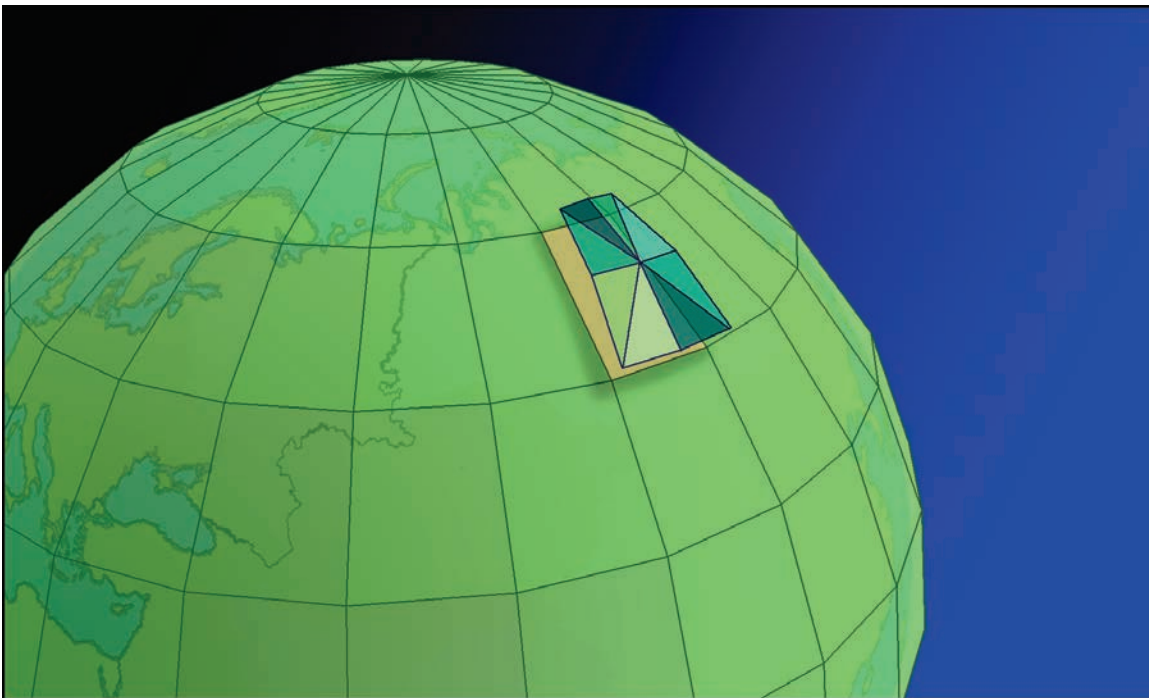
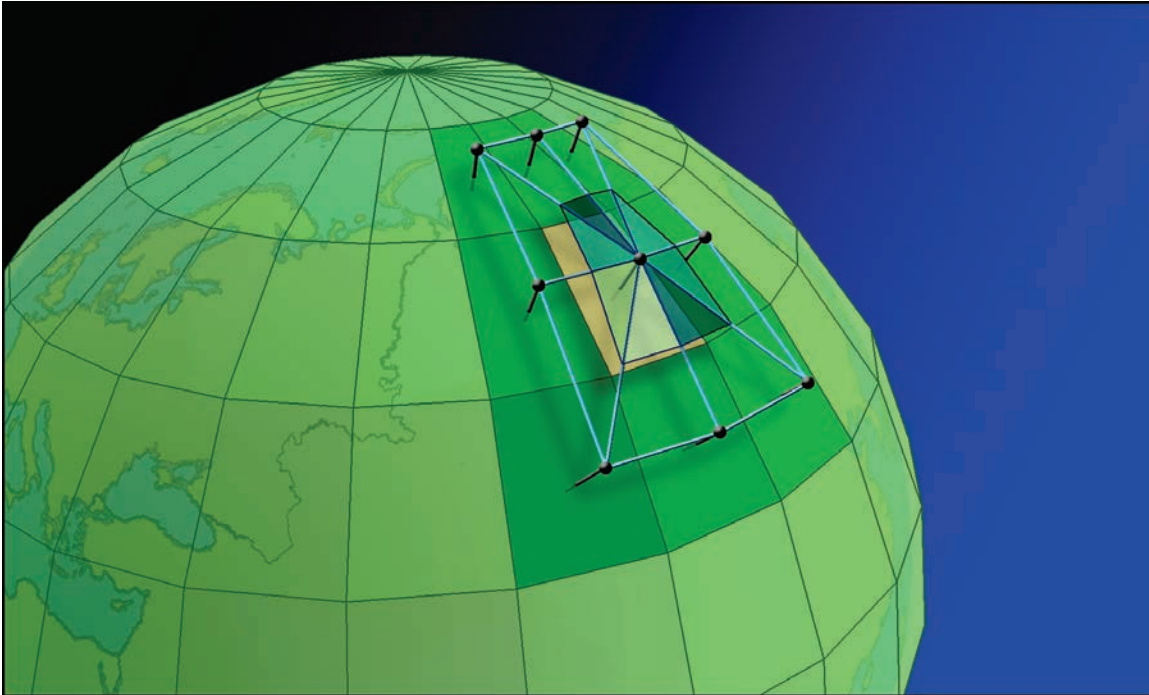
The surface area for a cell is based on the elevation in that cell, plus the elevations in the 8 cells around it.



3-dimensional triangles are generated (conceptually, at least; no actual vector objects are used in the calculations) from the centerpoints of the cells.



Triangles are clipped to the region covered by the central cell.



Technical Note: When analyzing a geographic raster, this tool treats the cell as a flat plane as illustrated in the images above. It does not consider the surface of the cell to be curved. Planimetric area is therefore equal to the area of the trapezoidal boundary of the cell, and surface area will always be \geq planimetric area.

ACCURACY ANALYSIS: TIN VS. RASTER SURFACE AREA COMPARISON

Note: This analysis was actually done using the ArcView 3.x version of the Surface Areas and Ratios extension (see http://www.jennessent.com/arcview/surface_areas.htm). The author has confirmed that the outputs of these extensions are identical, so the discussion below is simply adapted from the manual for the previous extension.

Summary

I compared the output of the Surface Area and Surface Ratio rasters derived by methods used in this extension with surface areas and ratios derived from a TIN-based analysis using the ESRI's 3D Analyst extension (ESRI 2000) and the author's Surface Tools for ArcView 3.x. extension (Jenness 2001). I computed the surface areas and ratios within 650 circular polygons ranging in size from 100m radius (approximately 3 raster cells) to 20,000m radius (approximately 146,454 raster cells).

Surface Ratios: I found that surface ratios correlated very well at all scales. The lowest correlation between surface ratio values was 0.993 (circle radius = 100). Raster-based values tended to be very close to TIN-based values, never differing more than 1% from their corresponding TIN-based value. In almost all cases, both accuracy and precision increased as the size of the circle increased, so that the circles with the greatest numbers of raster cells tended to be the most accurate and precise.

Surface Areas: Surface area values were extremely close between TIN-based methods and these raster-based methods, with correlations ranging from 0.993 to 1.000. Raster-based values tended to be higher than their TIN-based counterparts at low cell counts (< 264), but accuracy greatly increased at cell counts higher than 234 (circle radius = 800m). For polygons with \geq 264 cells, raster-based values were almost identical to TIN-based values and both accuracy and precision were nearly perfect.

The overall accuracy of this method, based on several additional analyses, can be also reviewed in:

[Jenness, J. S. 2004.](#) Calculating landscape surface area from digital elevation models. *Wildlife Society Bulletin*. 32(3):829-839

Methods

I tested the output accuracy of this extension by generating surface areas and ratios for 650 circular polygons in 7 different sizes and comparing results between a TIN-based method and the raster-based method used by this extension. These 650 polygons were randomly distributed across an elevation raster and all the polygons were completely within the elevation raster boundaries (i.e. none of the polygons overlaid any "No Data" areas). For a control group, I used the [Surface Tools extension](#) (Jenness 2001) to calculate what I assume to be the most accurate surface area and surface ratio values possible from the elevation data. I used a Z-factor = 0 (meaning that the new TIN will go through all the elevation points and therefore be as complex as possible. See Surface Tools manual for an explanation).

I then generated surface areas and ratios using the ArcView 3.x version of this extension. I compared the raster-based surface areas with the TIN-based surface areas for each of the 650 circular polygons using scatterplots, correlation statistics, and descriptive statistics. I used SPSS 9.0 to generate the statistics.

Polygon radii are in meters and rasters had a cell size of approximately 92.6 meters. The means, standard deviations and ranges reflect the ratios of (raster-based surface area) divided by (TIN-based surface area), or the ratio of (raster-based surface ratio) divided by (TIN-based surface ratio).

Results

In all cases, surface ratio values were very close between the TIN- and raster-based calculations. Accuracy and precision increased with higher cell counts, but even in the worst case the raster-based calculation was only 1% different than the TIN-based calculation.

Surface area values were always highly correlated, but raster-based calculations tended to be higher than TIN-based calculations at low cell counts. Surface area values for the 100m circles, with 4 raster cells each, tended to be over 9% higher when calculated from the raster. Raster-based surface areas for circles with up to 60 cells tended to be about 2.5% higher than their corresponding TIN-based surface areas.

Surface areas for circles with cell counts ≥ 234 tended to be almost identical between the raster- and TIN-based calculations, and correlations were almost perfect.

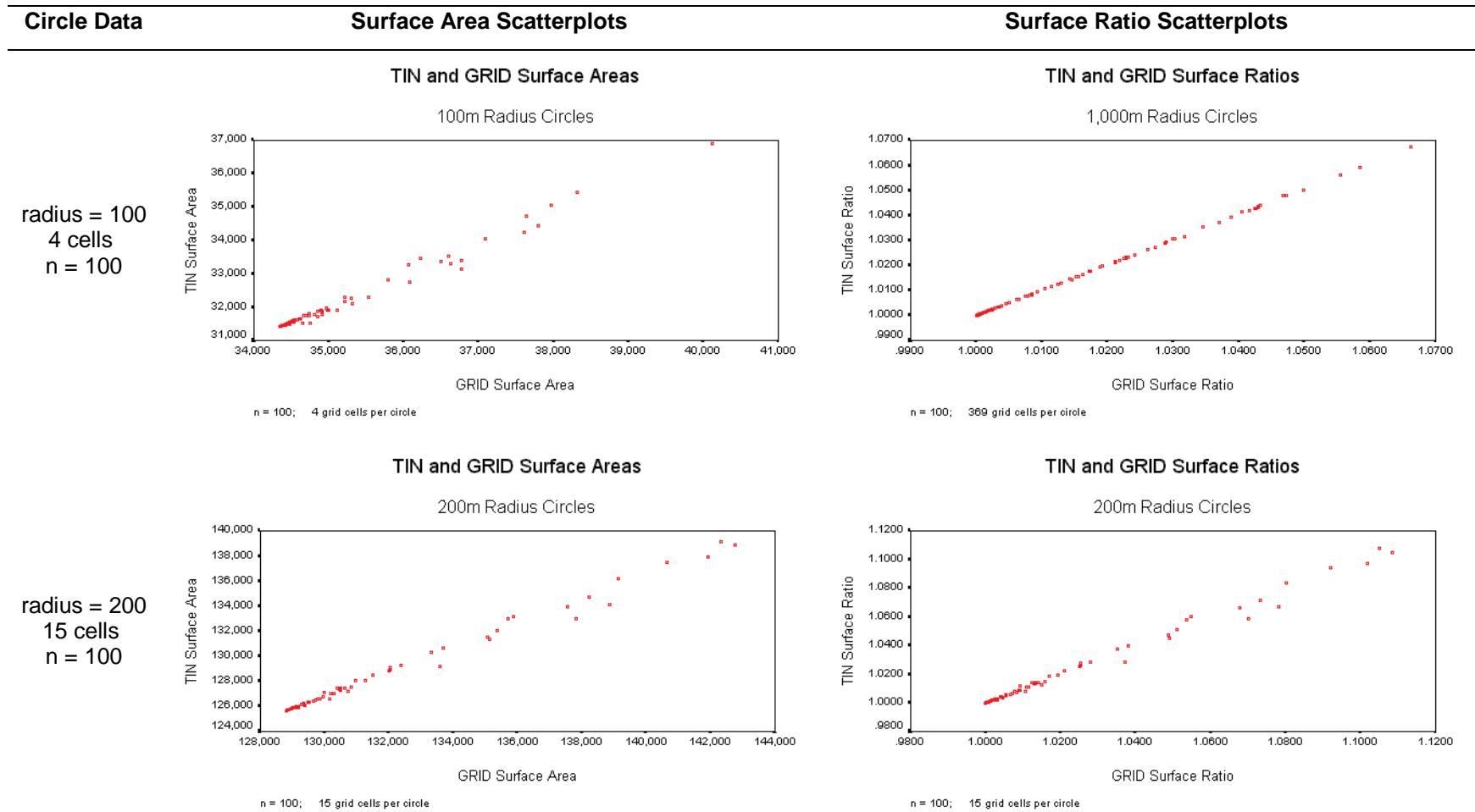
Table 1: Surface Area Comparisons - TIN vs. Raster-Based Output

Circle Radius	n	Avg. # Raster Cells in Polygon	Surface Area Comparison			
			Pearson Correlation	(RASTER Surface Area) / (TIN Surface Area)		
				mean	sd	range
100	100	4	0.993	1.0937	0.0039	1.08 - 1.11
200	100	15	0.996	1.0252	0.0022	1.02 - 1.04
400	100	60	0.999	1.0249	0.0011	1.02 - 1.03
800	100	234	1.000	0.9991	0.0004	1.00 - 1.00
1,000	100	369	1.000	1.0084	0.0003	1.01 - 1.01
2,000	100	1,465	1.000	1.0008	0.0003	1.00 - 1.00
20,000	50	146,361	1.000	0.9996	0.0003	1.00 - 1.00

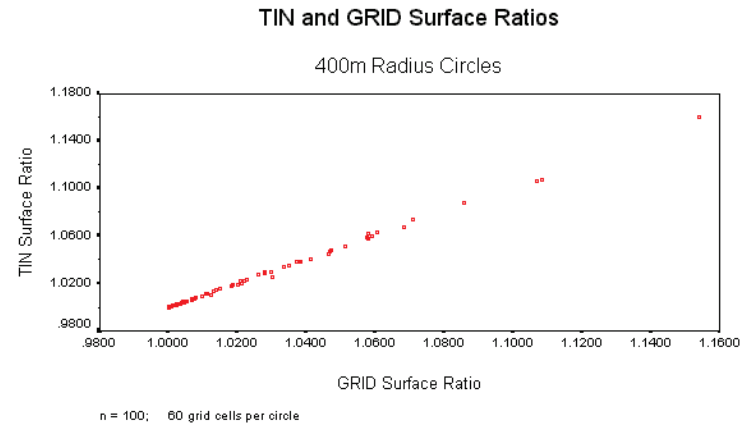
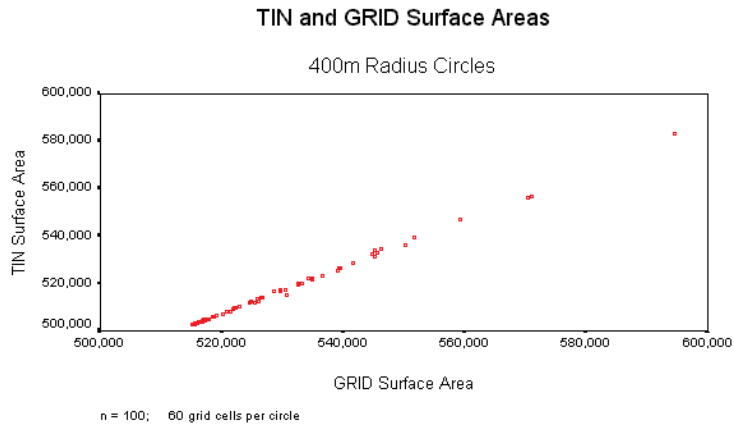
Table 2: Surface Ratio Comparisons - TIN vs. Raster-Based Output

Circle Radius	n	Avg. # Raster Cells in Polygon	Surface Ratio Comparison			
			Pearson Correlation	(RASTER Surface Ratio) / (TIN Surface Ratio)		
				mean	sd	range
100	100	4	0.993	1.0004	0.0036	0.99 - 1.01
200	100	15	0.996	1.0002	0.0021	1.00 - 1.01
400	100	60	0.999	0.9999	0.0011	0.99 - 1.00
800	100	234	1.000	0.9998	0.0004	1.00 - 1.00
1,000	100	369	1.000	0.9998	0.0003	1.00 - 1.00
2,000	100	1,465	1.000	0.9998	0.0003	1.00 - 1.00
20,000	50	146,361	1.000	0.9995	0.0002	1.00 - 1.00

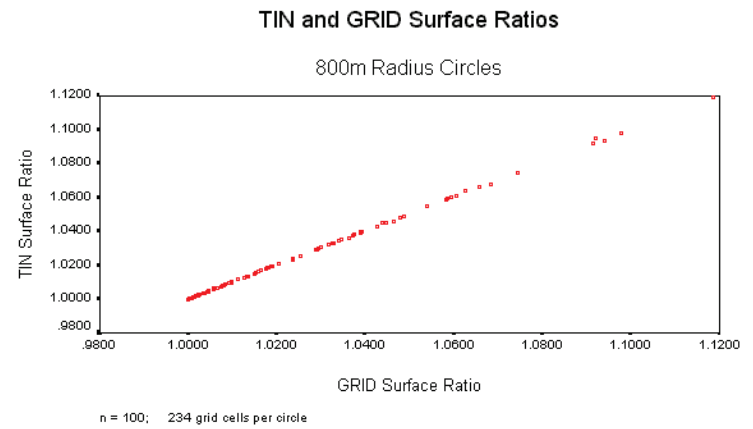
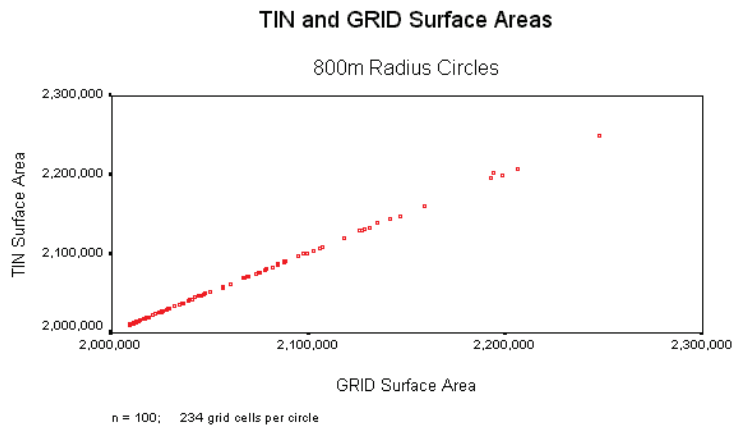
Table 6: TIN vs. Raster-Based Comparison Scatterplots



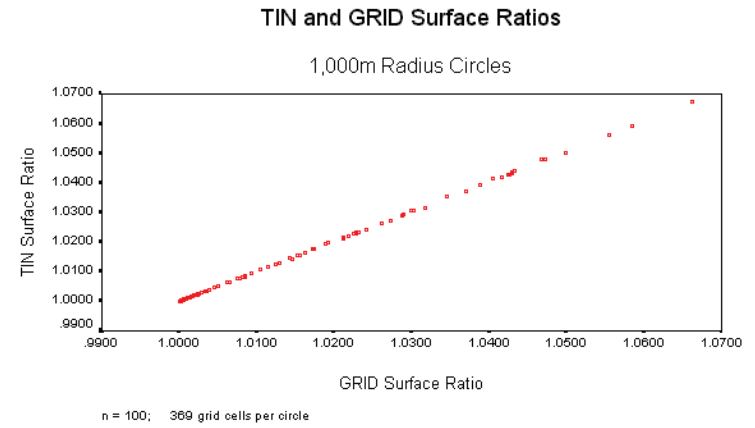
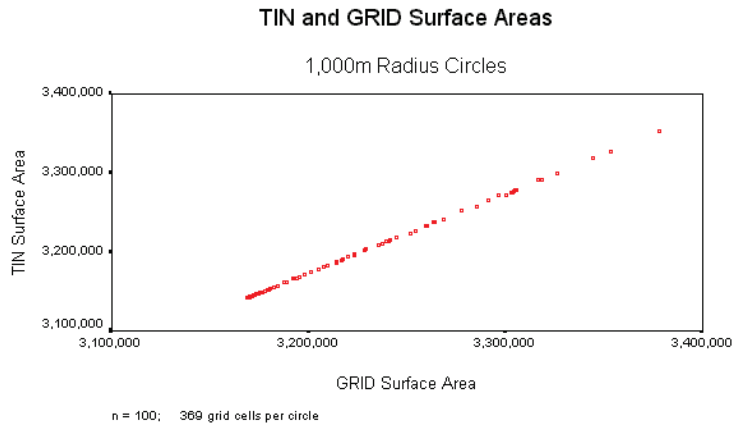
radius = 400
60 cells
n = 100



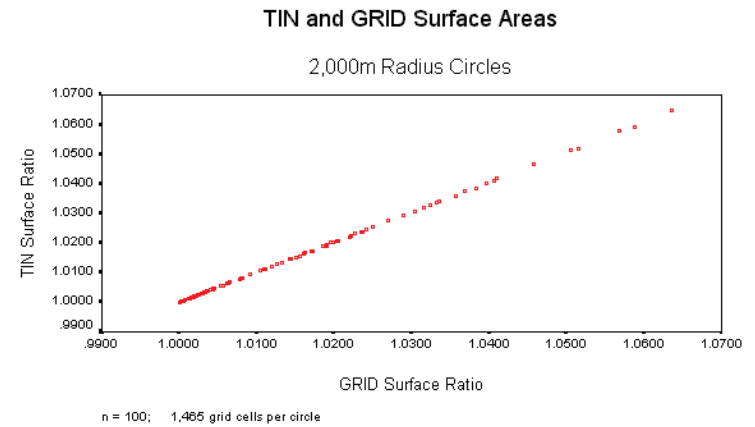
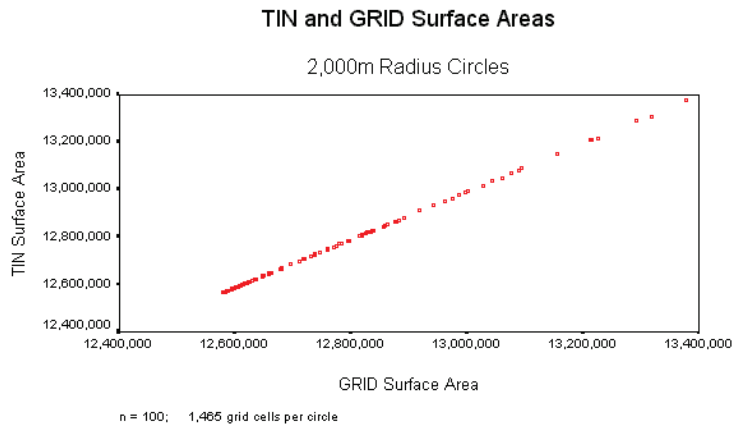
radius = 800
234 cells
n = 100



radius = 1,000
369 cells
n = 100

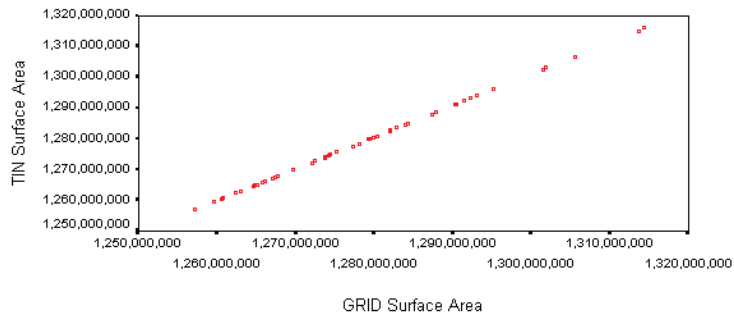


radius = 2,000
1,465 cells
n = 100



TIN and GRID Surface Areas

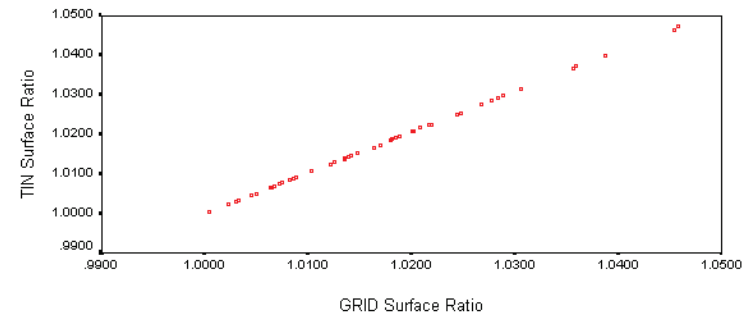
20,000m Radius Circles



n = 50; 146,361 grid cells per circle

TIN and GRID Surface Ratios

20,000m Radius Circles



n = 50; 146,361 grid cells per circle

radius =
20,000
146,361 cells
n = 50

Discussion

This extension appears to generate accurate surface area and surface ratio rasters. A cell value in a surface area raster appears to accurately reflect the surface area within the boundaries of that cell.

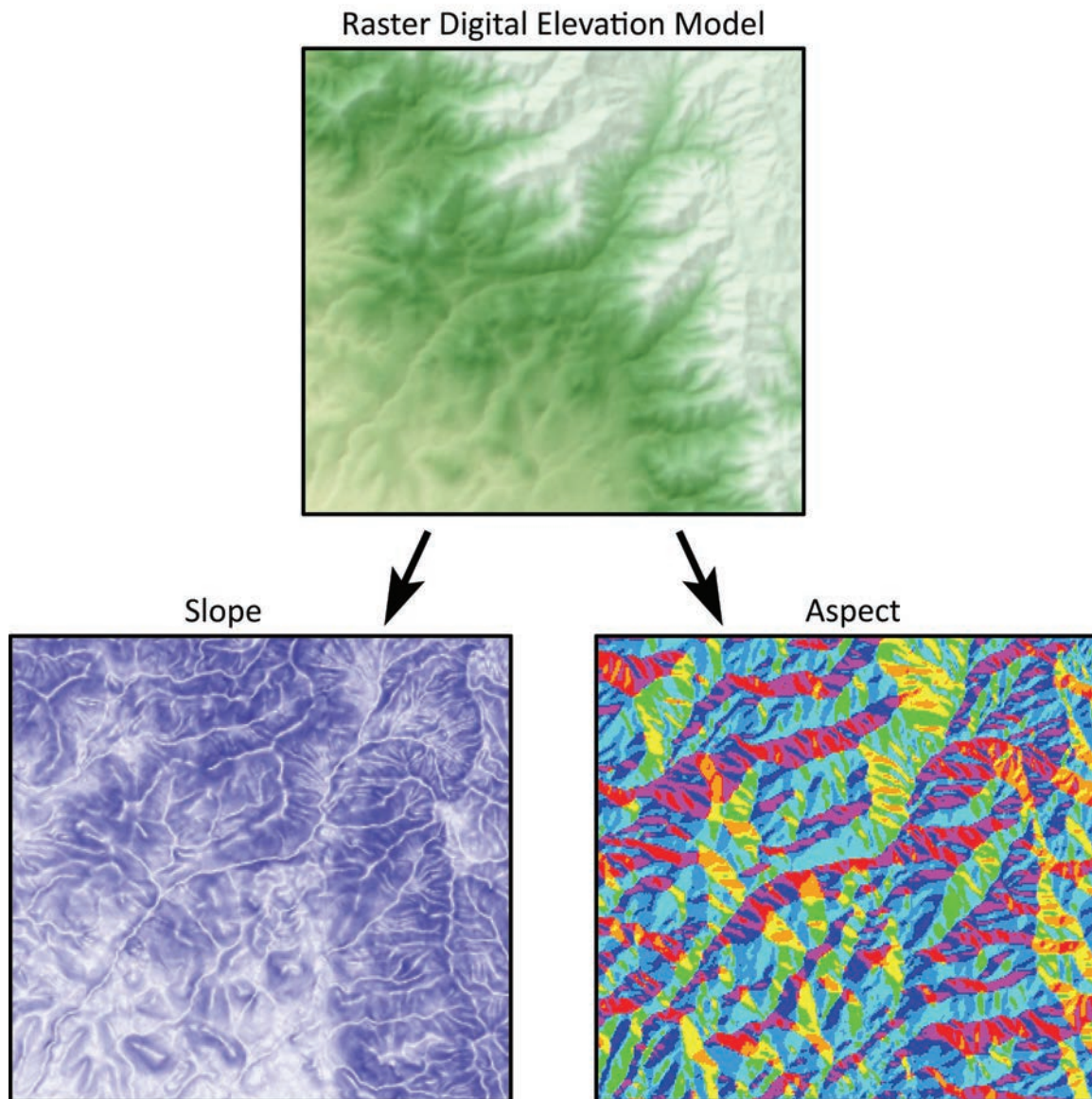
Surface areas within polygons can be accurately measured with this method provided the polygon contains a sufficient number of cells. With the circular polygons used in this analysis, accuracy and precision tended to be very high when cell counts equaled or exceeded 264.

Cumulative surface area and ratio accuracy within polygons is variable when the polygon is small, although most of this variation is likely due to the fact that identically-sized circles randomly placed on a raster will not generally have the same number of cells in each circle. A cell is considered to be included in a circle if that cell's center is located within the circle boundary, and slight deviations in position can cause large numbers of cells to be included or excluded. Accuracy is consistently low in cases of low cell counts, with raster-based values tending to be significantly higher than TIN-based values.

Recommendations

I feel that this extension does produce reliable surface area and surface ratio values from elevation rasters. Summary statistics within polygons is most accurate when the polygon includes a large number of cells (≥ 264), and accuracy is variable below that number.

Slope and Aspect:



SLOPE AND ASPECT BACKGROUND:

The slope of the landscape is a key variable in a wide variety of landscape analysis. It affects where structures or trails can be built, crops can be planted or harvested, the speed of flowing water and consequent erosion, landslide potential, and the list just goes on and on. Aspect is an important contributor to vegetation and habitat type, as north-facing slopes often have very different conditions and temperatures than south-facing slopes, and is also important in determining whether significant features are visible from a particular point on the landscape.

Slope and Aspect are both calculated from the directional East-West and North-South gradients. There are various methods for estimating these directional gradients from a 3x3 set of elevation points, but in general slope is defined as the gradient in the direction of maximum slope, and aspect is defined as the direction of maximum slope. Given that G = east-to-west gradient and H = north-to-south gradient, then slope and aspect are calculated as follows:

$$\% \text{ Slope} = \sqrt{G^2 + H^2} \quad (\text{Sometimes multiplied by } 100)$$

$$\text{Degrees Slope} = \text{atan} \sqrt{G^2 + H^2} \left(\frac{180}{\pi} \right)$$

$$\text{Mathematical Direction (in degrees)} = \text{atan } 2(H, -G) \left(\frac{180}{\pi} \right)$$

Mathematical direction is different than compass direction. In the mathematical polar coordinate system, 0 degrees is equivalent to due east and polar directions increase in a counter-clockwise direction. Compass direction starts at north and increases in a clockwise direction. Therefore we must convert the mathematical direction to a compass aspect according to the following rules:

if $\text{Direction} > 90^\circ$ then $\text{Aspect} = (450 - \text{Direction})$

Otherwise $\text{Aspect} = (90 - \text{Direction})$

By convention, slope is usually reported as a positive value even though it is technically a negative value in the aspect direction (Zevenbergen & Thorne, 1987).

This tool offers three methods to calculate slope and aspect, labeled following nomenclature by Jones (1998) as the *4-Cell Method*, *Horn's Method* and *Sharpnack & Akin's Method* (citations below). All tools work on both projected and unprojected raster DEMs. **Note:** ESRI's Spatial Analyst uses Horn's method.

Relative Accuracies of Methods:

These three methods, plus five additional ones not offered by this tool, were evaluated for accuracy by K. Jones (1998). In short, he found that the 4-cell method consistently was the most accurate, but only marginally better than Horn's method (in 2nd place) or Sharpnack & Akin's method (in 4th place).


Jones' analysis was performed on a mathematical surface in which the true exact slope and aspect at any point could be calculated. He then converted that surface to a raster DEM and calculated slopes and aspects using eight popular directional gradient calculation methods at a variety of spatial scales, and then compared the values with the true exact slopes and aspects to see which methods were most accurate. Jones then tested the methods on a true landscape DEM and found that the rankings between methods was consistent.

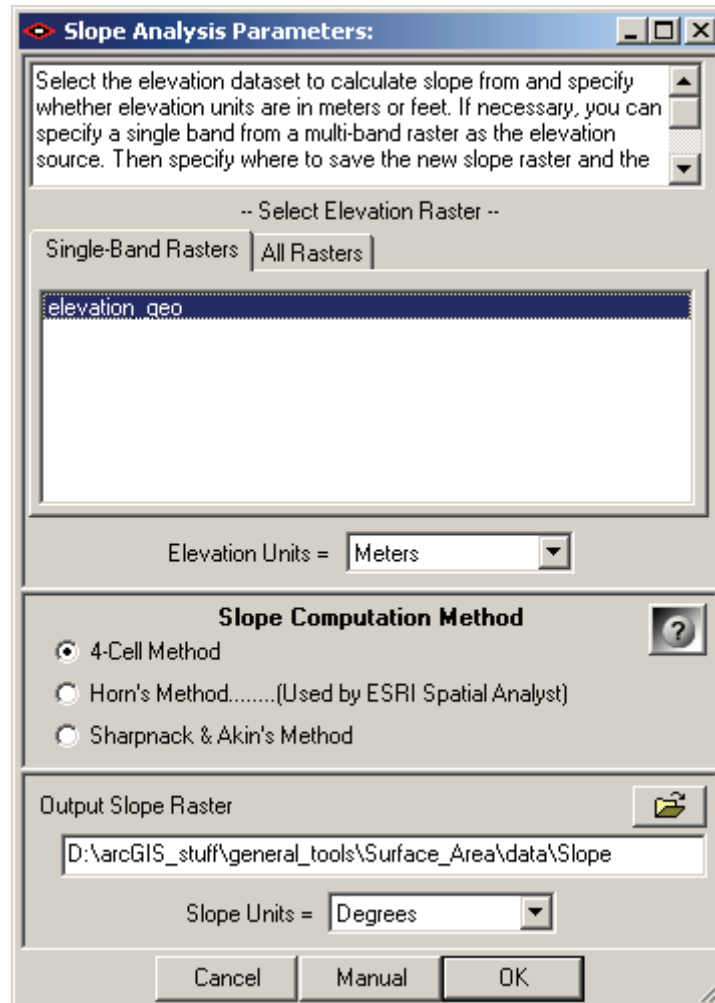
All three methods offered with this tool tended to produce very similar and consistently accurate results, and the rankings were always: 1st = 4-Cell Method, 2nd = Horn's Method and 4th = Sharpnack & Akin's Method. **Note:** The 3rd best performing method was a variation on Horn's method. In the interest of providing some variety of slope options, and because Sharpnack & Akin's equations are widely used in landscape curvature functions, I decided to include Sharpnack & Akin's method rather than Jones' 3rd-place contender.

What area is represented by the slope and aspect?

Michael Hodgson (1995) published a very interesting paper using the same mathematical surface that Jones (1998) did, in which he demonstrates that the area represented by computed slope and aspect is actually larger than the raster DEM cell size. 8-cell algorithms (such as *Horn's* or *Sharpnack & Akin's* methods) more accurately reflect values for an area approximately twice the cell size, and 4-cell algorithms (such as the *4-Cell* method) reflect an area approximately 1.6X the original cell area. For example, if you calculate slope or aspect from a raster with 30m cell size ($\pm 15\text{m}$ from the cell center), and you use either *Horn's* or *Sharpnack & Akin's* method, then your slope and aspect values will more accurately describe a region $\pm 21\text{m}$ from the cell center.

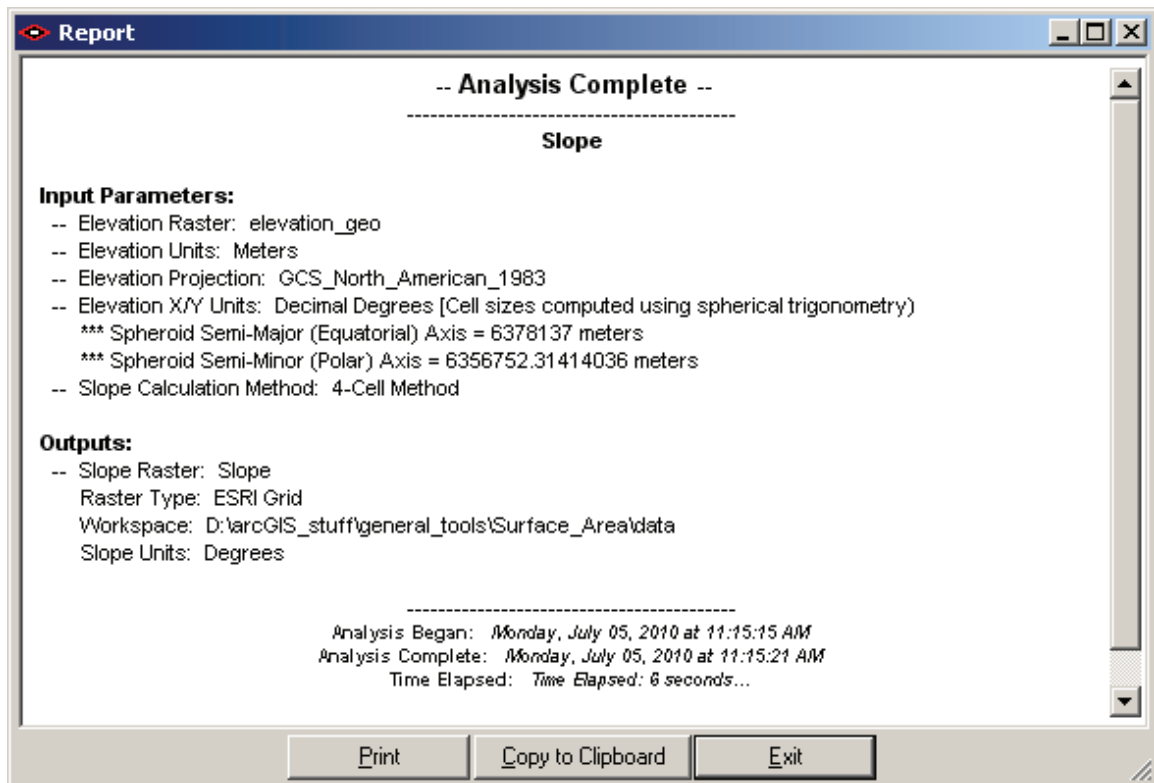
RUNNING THE SLOPE TOOL:

- 1) Load your elevation raster into ArcGIS.
- 2) Click the *Slope* button  to open the “Slope Analysis Parameters” dialog:




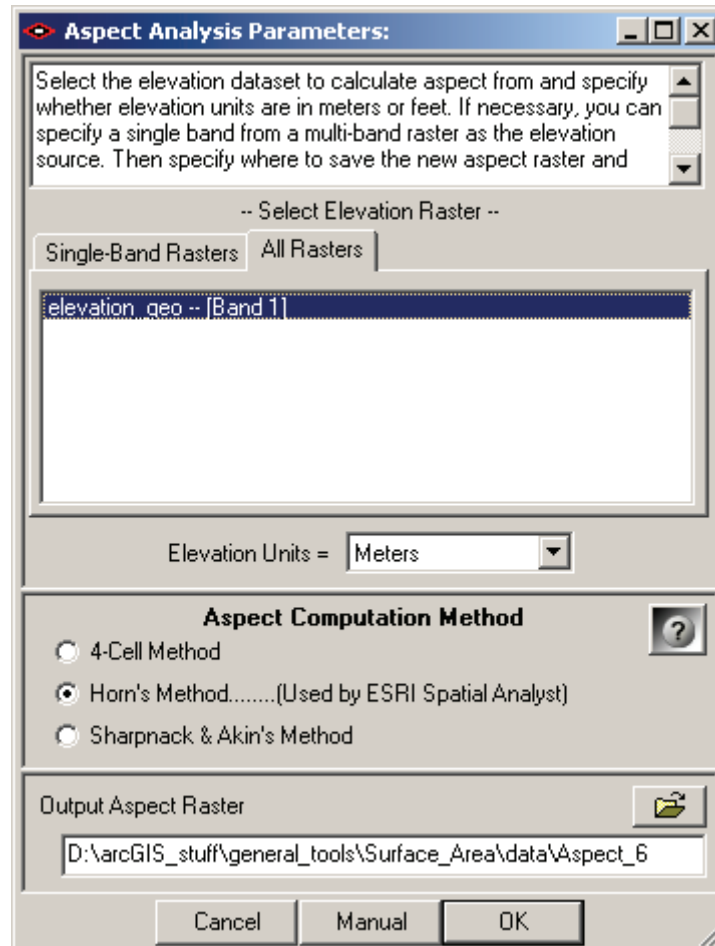
- 3) Select your elevation raster from the listbox of rasters in your map. If you wish to analyze a particular band of a multi-band raster, then click the “All Rasters” tab and select the specific band you wish to analyze.
- 4) Specify the elevation units of your dataset. The tool is currently written to only accept elevation values in meters or feet because those are by far the most common. Please contact the author (jeffj@jennessent.com) if you need additional elevation unit options.
- 5) Select which slope computation method you wish to use. These methods are described in detail in *Slope and Aspect Methods* below.
- 6) Specify where to save your slope raster.
- 7) Select your preferred slope units. Your options are:
 - a. **Degrees:** There are 90 degrees in a right angle.
 - i. Horizontal = 0 degrees
 - ii. A vertical cliff = 90 degrees.

- b. **Percent Slope:** Equal to $\frac{\Delta y}{\Delta x}$, or the rise over the run.
 - i. Horizontal = 0%
 - ii. A 45° slope = 100%
 - iii. A vertical cliff = infinite percent
 - c. **Gradians** (also known as gons or grades): There are 100 gradians in a right angle.
 - i. Horizontal = 0 gradians
 - ii. A vertical cliff = 100 gradians
- 8) Click 'OK' and the tool will generate your slope raster for you. Upon completion, it will provide you with a report detailing the analysis parameters:

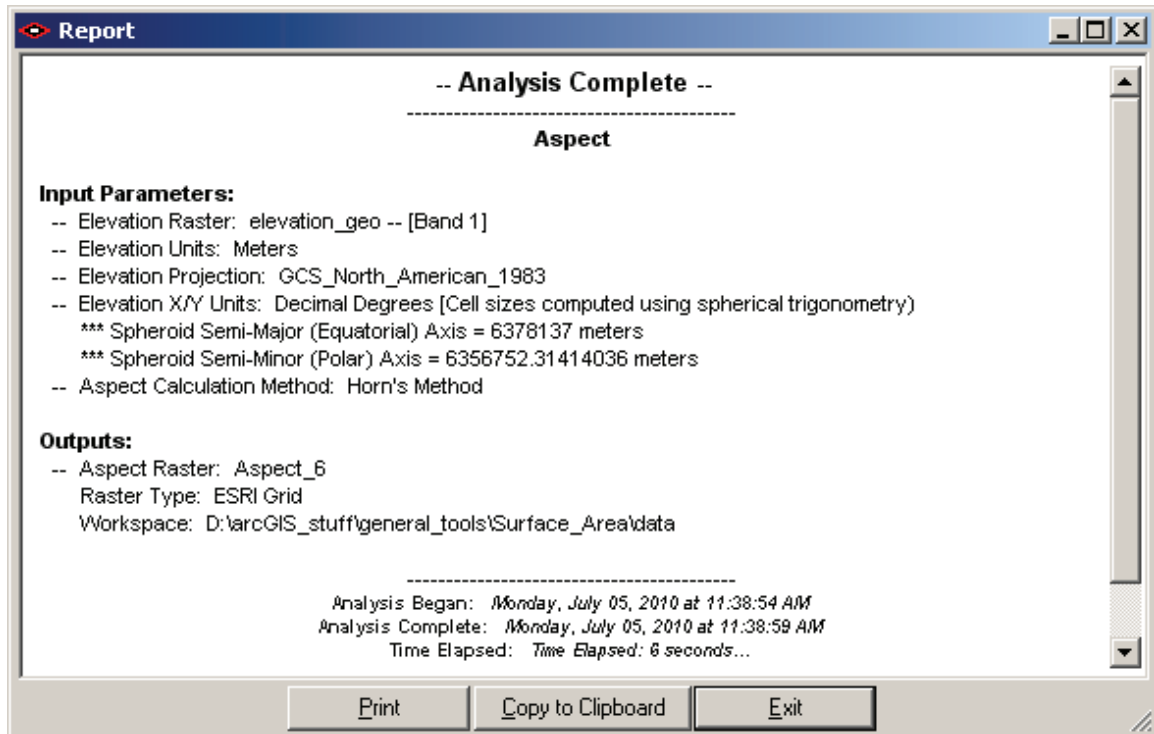


RUNNING THE ASPECT TOOL:

- 1) Load your elevation raster into ArcGIS.
- 2) Click the Aspect button  to open the “Slope Analysis Parameters” dialog:



- 3) Select your elevation raster from the listbox of rasters in your map. If you wish to analyze a particular band of a multi-band raster, then click the “All Rasters” tab and select the specific band you wish to analyze.
- 4) Specify the elevation units of your dataset. The tool is currently written to only accept elevation values in meters or feet because those are by far the most common. Please contact the author (jeffj@jennessent.com) if you need additional elevation unit options.
- 5) Select which slope computation method you wish to use. These methods are described in detail in *Slope and Aspect Methods* below.
- 6) Specify where to save your aspect raster.
- 7) Click 'OK' and the tool will generate your aspect raster for you. Upon completion, it will provide you with a report detailing the analysis parameters:




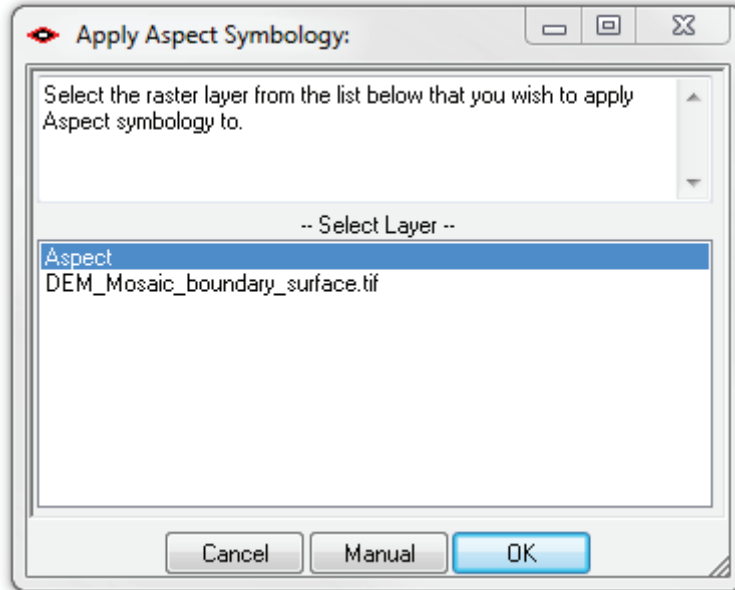
- 8) Your Aspect raster will be a floating point raster with values ranging from 0 to 360. To maintain consistency with the existing ArcGIS aspect tool, your new aspect layer will be symbolized as 8 categorical directions, plus an additional category for flat areas.

Aspect_6

Flat
North (0 - 22.5)
Northeast (22.5 - 67.5)
East (67.5 - 112.5)
Southeast (112.5 - 157.5)
South (157.5 - 202.5)
Southwest (202.5 - 247.5)
West (247.5 - 292.5)
Northwest (292.5 - 337.5)
North (337.5 - 359.7026)

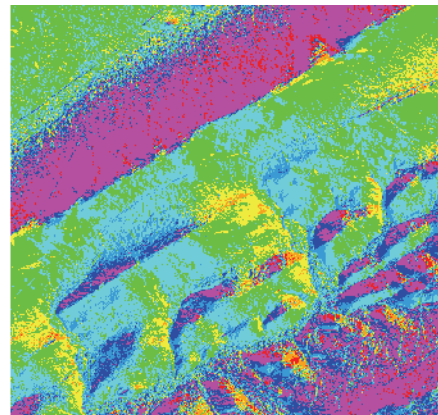
RUNNING THE ASPECT SYMBOLOGY TOOL:

- 1) Load your existing aspect raster into ArcGIS.
- 2) Click the *Apply Aspect Symbolology* button  to open the "Apply Aspect Symbolology" dialog:



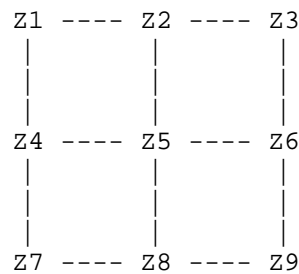
- 3) Simply select the raster layer containing your aspect values and click OK. Your new layer will be symbolized with the traditional 9-category directional symbology.

- Aspect_6
- Flat
 - North (0 - 22.5)
 - Northeast (22.5 - 67.5)
 - East (67.5 - 112.5)
 - Southeast (112.5 - 157.5)
 - South (157.5 - 202.5)
 - Southwest (202.5 - 247.5)
 - West (247.5 - 292.5)
 - Northwest (292.5 - 337.5)
 - North (337.5 - 359.7026)



SLOPE AND ASPECT METHODS:

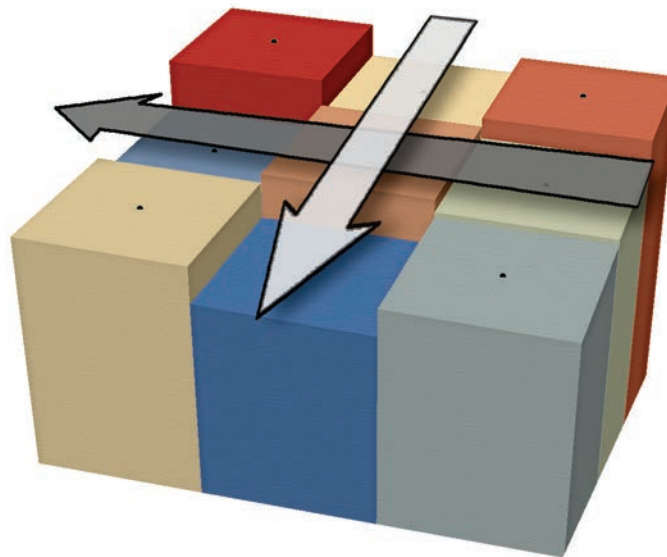
The three methods offered with this tool calculate east-west and north-south gradients from a 3x3 array of elevation points as follows:



- 1) **4-Cell Method** (Fleming & Hoffer [1979], Zevenbergen & Thorne [1987]): The north-south gradient is based on the cells immediately north and south of the central cell, and the east-west gradient is calculated from the cells immediately east and west of the central cell.

$$\text{East-West Gradient} = G = \frac{(-Z_4 + Z_6)}{2\Delta x}$$

$$\text{North-South Gradient} = H = \frac{(Z_2 - Z_8)}{2\Delta y}$$



Note: Hodgson (1995) found that this method actually described the slope and aspect in an area approximately 1.6 times the original cell size.

- 2) **Horn's Method** (Horn [1981], Burrough & McDonnell [1998]): The north-south gradient is calculated from all three columns in the 3x3 array, with the central column weighted twice as high as the two outer columns. The east-west gradient is calculated in a similar manner.

$$\text{East-West Gradient} = G = \frac{[(Z_3 + 2Z_6 + Z_9) - (Z_1 + 2Z_4 + Z_7)]}{8\Delta x}$$

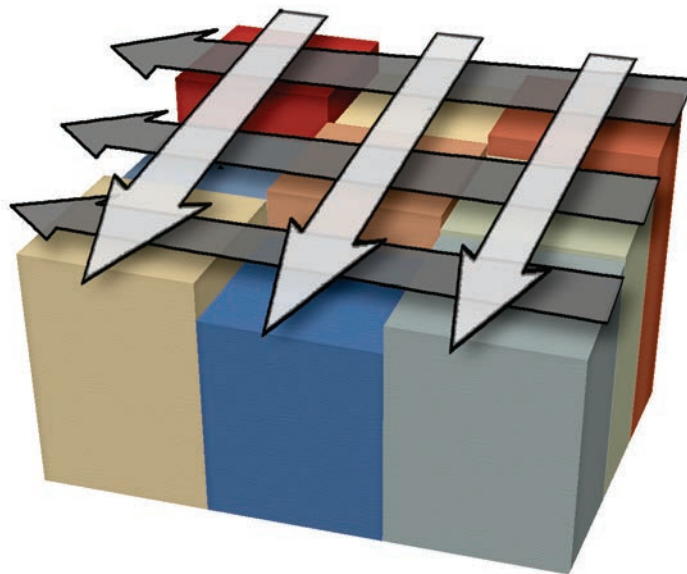
$$\text{North-South Gradient} = H = \frac{[(Z_1 + 2Z_2 + Z_3) - (Z_7 + 2Z_8 + Z_9)]}{8\Delta y}$$



Note: Hodgson (1995) found that this method actually described the slope and aspect in an area approximately 2 times the original cell size.

- 3) Sharpnack & Akin's Method** (Sharpnack & Akin [1969], Evans [1979], Florinsky [1998b]): The north-south gradient is calculated from all three columns in the 3x3 array, with all columns weighted equally. The east-west gradient is calculated in a similar manner.

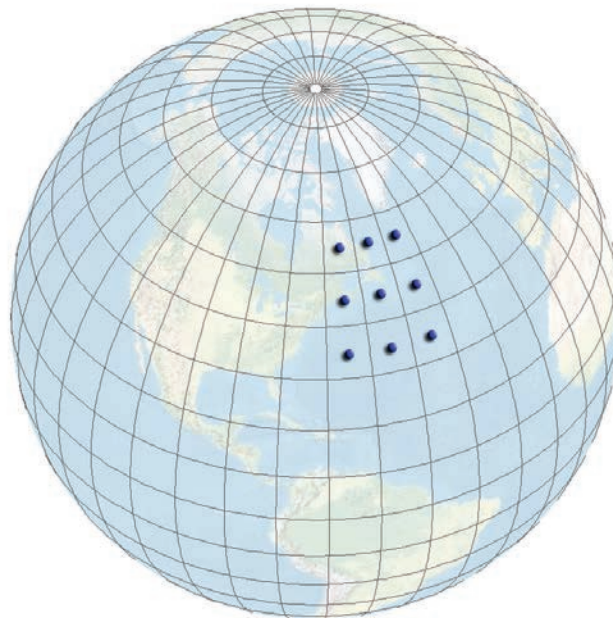
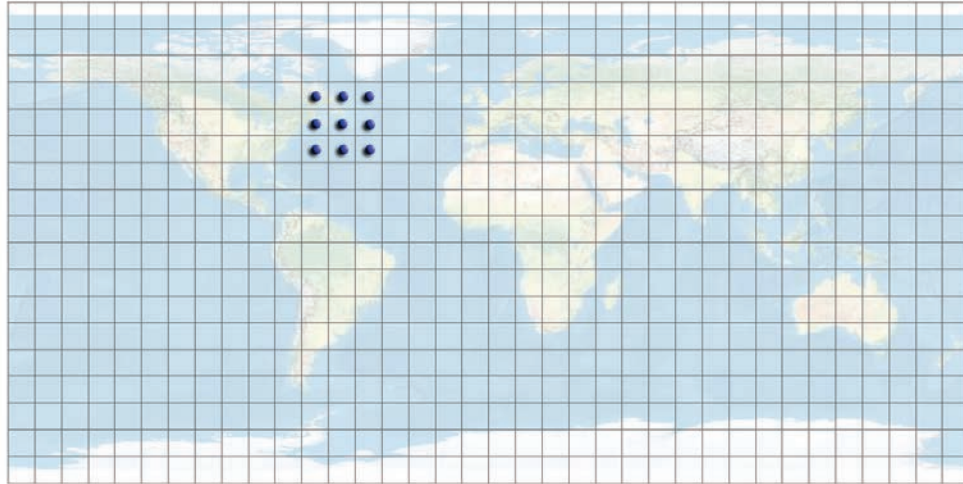
$$\text{East-West Gradient} = G = \frac{Z_3 + Z_6 + Z_9 - Z_1 - Z_4 - Z_7}{6\Delta x}$$
$$\text{North-South Gradient} = H = \frac{Z_1 + Z_2 + Z_3 - Z_7 - Z_8 - Z_9}{6\Delta y}$$



Note: Hodgson (1995) found that this method actually described the slope and aspect in an area approximately 2 times the original cell size.

Adjusting the Methods for Latitude / Longitude Data:

The equations above all work for 3x3 arrays in which the elevation locations are all equally spaced along both the X- and Y-axes. However, this is not the case with latitude / longitude data. Raster cells that are square in unprojected space actually form trapezoids when viewed on the actual surface of the planet.



In this case, we cannot treat the cell size as a single constant value. The distance between elevation points when moving north-to-south is almost certain to be different than the distance when moving east-to-west, and the east-to-west distance between points in one row will be different than the east-to-west distance in any other row. All that we can rely on is that, within a single row, the north-to-south distances will stay constant and the east-to-west distances will stay constant. All bets are off when we move to another row.

However, with two small caveats, we can still make the equations above work for us. The key is that we be able to calculate the east-west and north-south gradients separately. If we examine the equations for G and H , we see that they can be decomposed into gradients for individual rows and columns:

1) 4-Cell Method:

$$\text{East-West Gradient} = G = \frac{(-Z_4 + Z_6)}{2\Delta x} = \frac{\Delta Z_{(\text{Row } 2)}}{\text{Length}_{(\text{Row } 2)}}$$

$$\text{North-South Gradient} = H = \frac{(Z_2 - Z_8)}{2\Delta y} = \frac{\Delta Z_{(\text{Col } 2)}}{\text{Length}_{(\text{Col } 2)}}$$

If you can determine the length of row 2 and column 2, then you can calculate the gradients of each. It does not matter that the row and column distances are different than each other (Ritter, 1987). As Evans (1980) points out, the important factor is to make sure the East-West and North-South directional gradients represent the same point (but see caveat below). Fortunately the methods for calculating the distance between points on the spheroid are well-known and published (see Vincenty [1975] and Jenness [2009]), and this extension handles it automatically for you.

2) Horn's Method:

$$\begin{aligned} \text{East-West Gradient} = G &= \frac{[(Z_3 + 2Z_6 + Z_9) - (Z_1 + 2Z_4 + Z_7)]}{8\Delta x} \\ &= \frac{\Delta Z_{(\text{Row } 1)} + 2(\Delta Z_{(\text{Row } 2)}) + \Delta Z_{(\text{Row } 3)}}{\text{Length}_{(\text{Row } 1)} + 2(\text{Length}_{(\text{Row } 2)}) + \text{Length}_{(\text{Row } 3)}} \end{aligned}$$

$$\begin{aligned} \text{North-South Gradient} = H &= \frac{[(Z_1 + 2Z_2 + Z_3) - (Z_7 + 2Z_8 + Z_9)]}{8\Delta y} \\ &= \frac{\Delta Z_{(\text{Col } 1)} + 2(\Delta Z_{(\text{Col } 2)}) + \Delta Z_{(\text{Col } 3)}}{\text{Length}_{(\text{Col } 1)} + 2(\text{Length}_{(\text{Col } 2)}) + \text{Length}_{(\text{Col } 3)}} \end{aligned}$$

Horn's method basically takes a weighted average of all three rows (for east-west gradients) or columns (for north-south gradients), with the center row or column weighted twice as much as the outer ones. As with the 4-cell method, it is a simple matter to plug in the true row and column lengths into the equations.

3) Sharpnack & Akin's Method:

$$\begin{aligned} \text{East-West Gradient} = G &= \frac{Z_3 + Z_6 + Z_9 - Z_1 - Z_4 - Z_7}{\Delta x} \\ &= \frac{\Delta Z_{(\text{Row } 1)} + \Delta Z_{(\text{Row } 2)} + \Delta Z_{(\text{Row } 3)}}{\text{Length}_{(\text{Row } 1)} + \text{Length}_{(\text{Row } 2)} + \text{Length}_{(\text{Row } 3)}} \end{aligned}$$

$$\begin{aligned} \text{North-South Gradient} = H &= \frac{Z_1 + Z_2 + Z_3 - Z_7 - Z_8 - Z_9}{\Delta y} \\ &= \frac{\Delta Z_{(\text{Col } 1)} + \Delta Z_{(\text{Col } 2)} + \Delta Z_{(\text{Col } 3)}}{\text{Length}_{(\text{Col } 1)} + \text{Length}_{(\text{Col } 2)} + \text{Length}_{(\text{Col } 3)}} \end{aligned}$$

Sharpnack & Akin's method is similar to Horn's, except that each row and column is weighted equally. Again, we only need to plug in the true row and column lengths to get the correct east-west and north-south gradients.

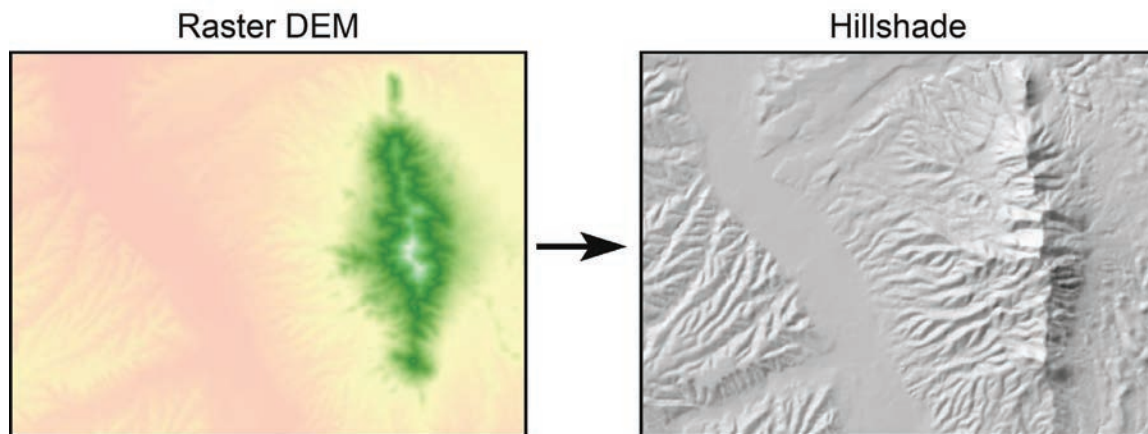
Caveats:

- 1) These modifications of the established equations assume that the 3x3 array of elevation points lie on a plane, which is not technically true. By definition, latitude and longitude coordinates imply that the locations are on the curved surface of a sphere or spheroid.

The modifications above do **not** account for the underlying curvature of the planet, but this is unlikely to be a significant problem in most cases. For a 1-ArcSecond resolution raster (approximately 30m cell size at the equator), the latitude / longitude coordinates deviate from a plane by only a few hundredths of a millimeter.

- 2) As Evans (1980) noted, these equations assume that the east-west and north-south gradients reflect the same point. The modifications above do not precisely give gradients for exactly the same points. More specifically, the east-west gradients reflect the line connecting the first and last points in the row, but that line does not quite intersect the point in the center cell (except in the rare case that the line is exactly on the equator). Because the points in rows follow a constant latitude, they therefore by definition follow a curve because parallels of latitude constantly curve toward the closest pole (except for the equator). **However**, I argue that this error is negligible in most landscape analyses. In a 1-ArcSecond raster, at 35° latitude, this central point deviates from the line connecting the two outer points by approximately the width of a human hair (about 0.00006m). Even with a 1-km pixel raster, the central cell only deviates from a straight line by 0.2m. This error could be mathematically corrected, but it is so far below the general horizontal and vertical error level of any DEM that I know of that it is not worth worrying about.

Hillshading



BACKGROUND:

Hillshading is an extremely useful way to depict the topographic relief of a landscape. Few methods are as intuitive and easy to understand as a hillshade. A good hillshade lets you understand immediately what areas are ravines, ridges, peaks or valleys. For those who are interested in this topic, Horn (1991) describes in detail the history, logic and algorithms of a great number of hillshading methods, including methods to incorporate different reflectance levels of the landscape.

Eduard Imhof wrote a classic book on the topic (“Cartographic Relief Presentation”, originally published in German in 1965, translated into English and expanded in 1982, and now available in a 2007 edition from ESRI press; see Imhof [2007]). Imhof’s book also goes into many more general methods of making clear and understandable maps and is a wonderful cartographic reference. If you are interested in the mathematical algorithms of hillshading, then Horn (1991) goes into much more detail than Imhof and, in fact, Imhof actually argues against computer automation of hillshading. He believed the only way it could be done accurately was manually by a competent cartographer. Computer-generated hillshades were only acceptable as a working tool, and only then if the methods were “time-saving, efficient and graphically faultless.” He was especially concerned with the fact that programmed algorithms were essentially blind to the landscape, and therefore could not consider the context of a slope when shading it (See Imhof [2007], p. 209 – 212). For example, topographic features that lay along the line of illumination could disappear and objects that lay perpendicular to the line could be overly accentuated. Of course he was writing in 1982, and probably did not have many high-quality samples of computer-generated hillshades to judge from.

The “Swiss Method”:

Various authors have suggested modifications to the basic hillshade. One of the more popular methods is generally referred to as the “Swiss Method”, after Imhof and his contemporaries. Strictly speaking, the “Swiss Method” is a manual method where the cartographer draws the hill shading by hand. Kurt Brassel introduced some automated methods to replicate some of the best qualities of the hand-drawn Swiss method (Brassel 1974), specifically recommending three general modifications.

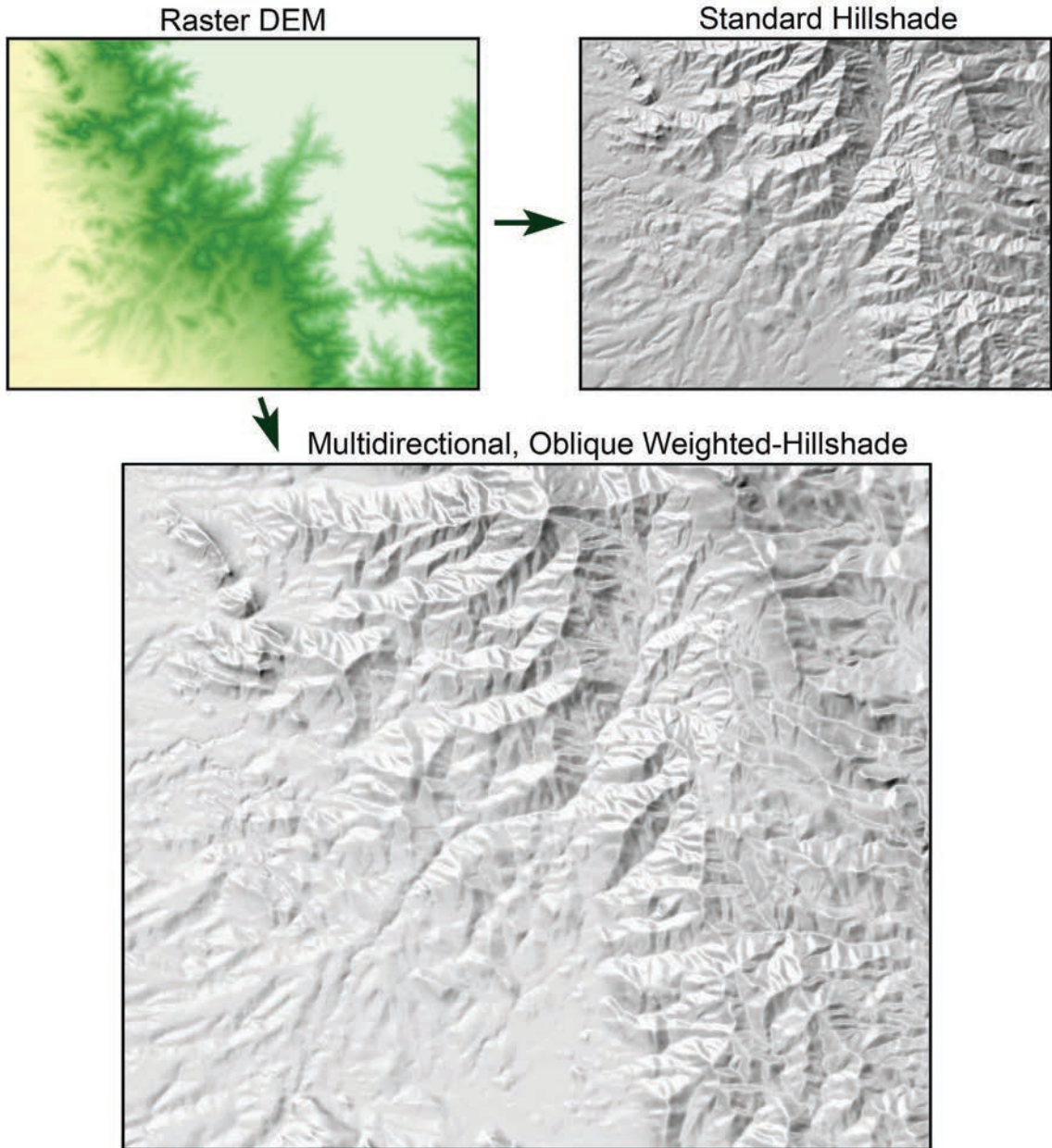
- 1) *Introduce light sources from multiple directions:* Brassel recommended setting a variable light source direction for each cell, based on a primary illumination direction \pm a directional adjustment computed from the aspect of that cell. Brassel also recommended that the illumination inclination angle (i.e. how high the sun is) for each raster cell should be variable, based on the deviation of the surface normal to the primary illumination source. By Brassel’s method, every cell could receive a hillshade based on a different illumination angle. Imhof (2007, p. 211) suggests a similar but simpler method, where the

- hillshade is calculated from two stationary light sources separated by a small amount ($10^\circ - 15^\circ$).
- 2) *Atmospheric Adjustments*: Brassel suggested that both the contrast and the brightness of the hillshade should decrease at lower elevations, and increase at higher elevations, in order to mimic atmospheric effects. The contrast could be adjusted by modifying the reflectance based on the elevation, such that low elevations would have relatively little reflectance. Brightness could be adjusted simply by darkening the cell according to the relative elevation.
 - 3) *Blur the final hillshade*: Brassel felt that small topographic features (misnamed “small-scale” in his manuscript) tended to have less information to describe them, and that consequently the hillshading of such features was more likely to be “fictive’ or ‘creative’”. Therefore such small features should be blurred to de-emphasize them. He also was working with dot-matrix printers to print his hillshades, and he recommended photographically blurring the output to increase the aesthetic quality of the hillshade. Others advocate blurring the hillshade if you simply want it to be a little less prominent or noticeable in the map (see especially Buckley & Barnes, 2004).

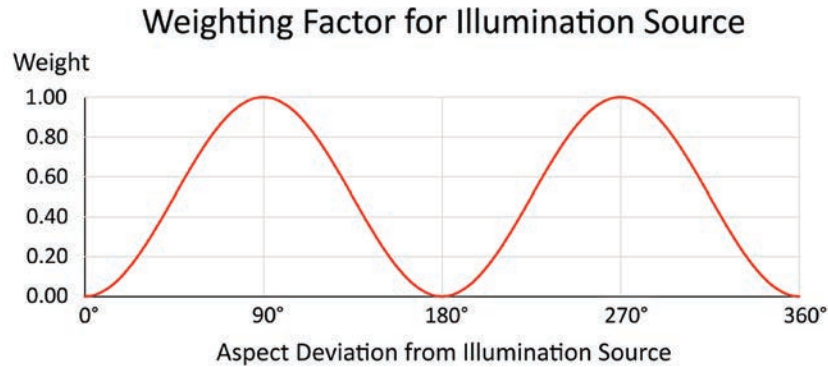
How this extension replicates some “Swiss Method” functions:

This extension does not offer up a full “Swiss Method” menu, but it does offer some modifications to the basic hillshade based on Brassard’s manuscript.

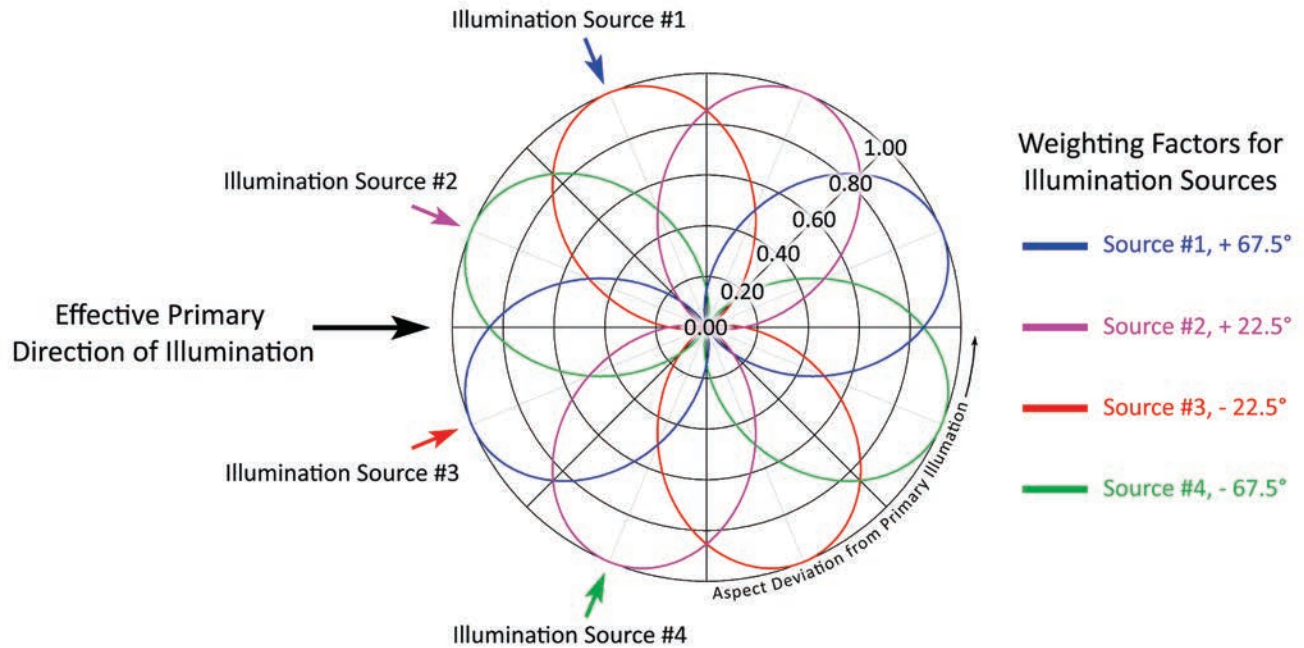
- 1) *Light Sources from Multiple Directions*: This tool currently does not offer a function to specify multiple stationary sources of illumination, nor does it offer Brassel’s variable direction and inclination for every cell. However, it does offer Robert Mark’s *Multidirectional, Oblique-Weighted* (MDOW) method (Mark 1992). By this method the landscape is illuminated from 4 directions, each separated by 45° and covering a range of 135° . The hillshade generated by each of these illumination sources is then weighted according to the aspect of that cell.



Interestingly, the weight of each source of illumination is highest at 90° from that source angle and lowest (actually weighted 0) directly facing the illumination.



The cumulative effect produces a hillshade illuminated generally from the average of the 4 illumination directions. The graph below (adapted from Mark's original) shows the weighting for all 4 illuminations sources based on the deviation of the cell's aspect value from the cumulative direction of illumination.



- 2) *Atmospheric Adjustments*: This tool currently does not offer a function to modify landscape reflectance (i.e. contrast) based on the relative elevation. However, it does offer the option for hypsometric tinting with both the standard and MDOW hillshade tools. This hypsometric tinting allows you to darken the lower elevations.

The tool allows you to set the strength of the tint, where the Strength value is the maximum possible brightness of the cell with the lowest elevation. A value of 100 means that the cell with the lowest elevation will be black (i.e. have a hillshade value = 0) regardless of how directly it faces the sun. A value of 50 means that the lowest cell will be 50% as bright as it would be without any hypsometric shading, and a value of 0 will mean that no hypsometric shading is applied.

$$\text{Shading Factor} = 1 - \left(\frac{Z_0 - Z_{\min}}{Z_{\max} - Z_{\min}} \right) \left(\frac{S}{100} \right) - \left(\frac{100 - S}{100} \right)$$

where :

Z_0 = Elevation of focal cell

Z_{\min} = Minimum Elevation

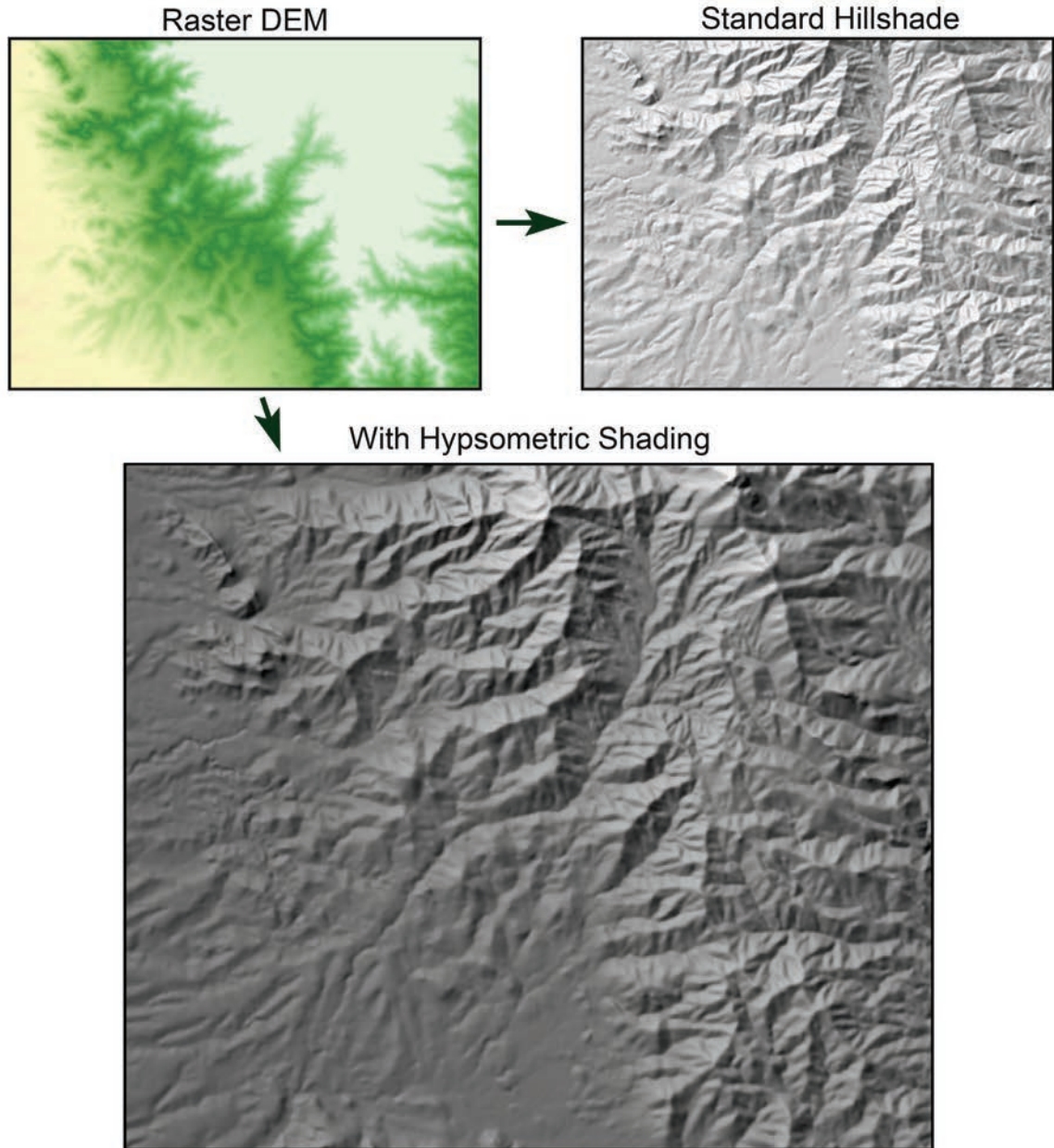
Z_{\max} = Maximum Elevation

S = Hypsometric Strength, $0 \leq S \leq 100$

This equation produces a hypsometric tinting factor between 0 and (Strength / 100) for all raster cells, where 0 applies to the highest cell and (Strength / 100) applies to the lowest cell.


This hypsometric tint value is subtracted from 1 and multiplied by the actual hillshade (H) value to produce the hypsometrically shaded hillshade (HSH) value:

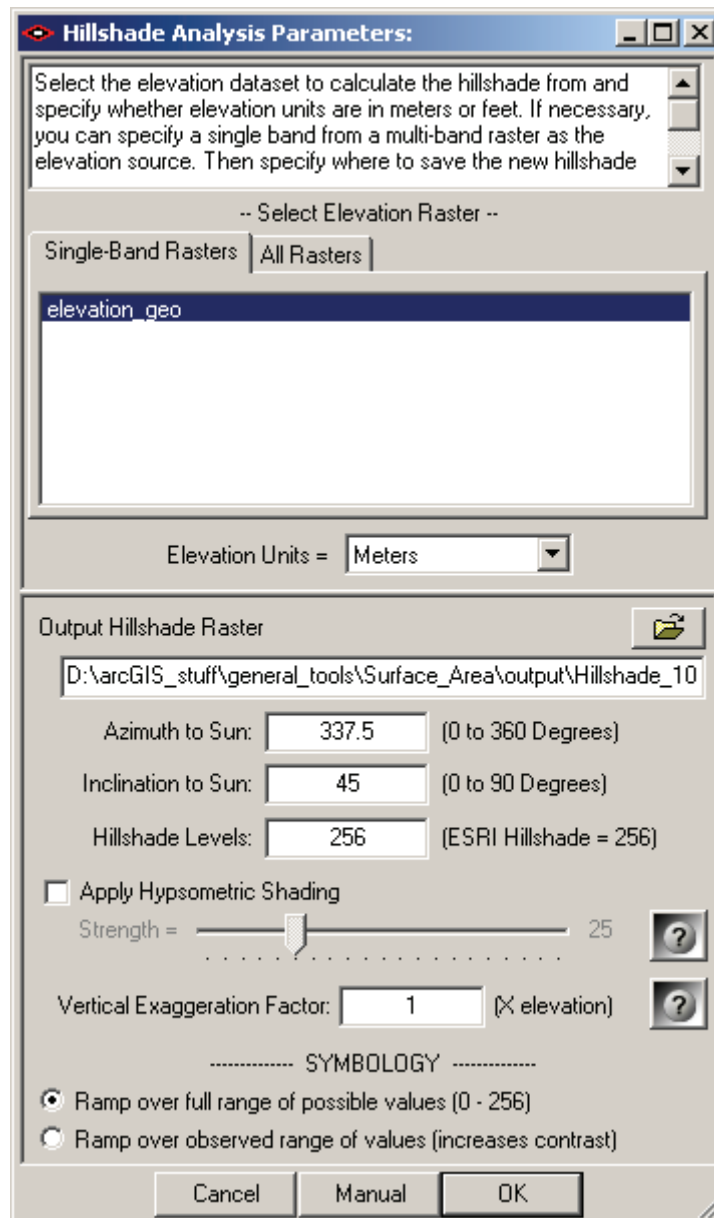
$$HSH = (1 - \text{Shading Factor}) * H$$



- 3) *Blur the hillshade*: This tool does not offer any method to blur the hillshade. If this is a function you would like to apply, then it is simple enough to blur the hillshade yourself using the Spatial Analyst Neighborhood Statistics function to calculate the neighborhood mean of the hillshade. You can do this to your hillshade after creating it, or you can calculate the neighborhood mean of the original DEM prior to calculating the hillshade. Either method will blur your hillshade.

USING THE STANDARD HILLSHADE TOOL:

- 1) Load your elevation raster into ArcGIS.
- 2) Click the *Hillshade* button  to open the “Hillshade Analysis Parameters” dialog:



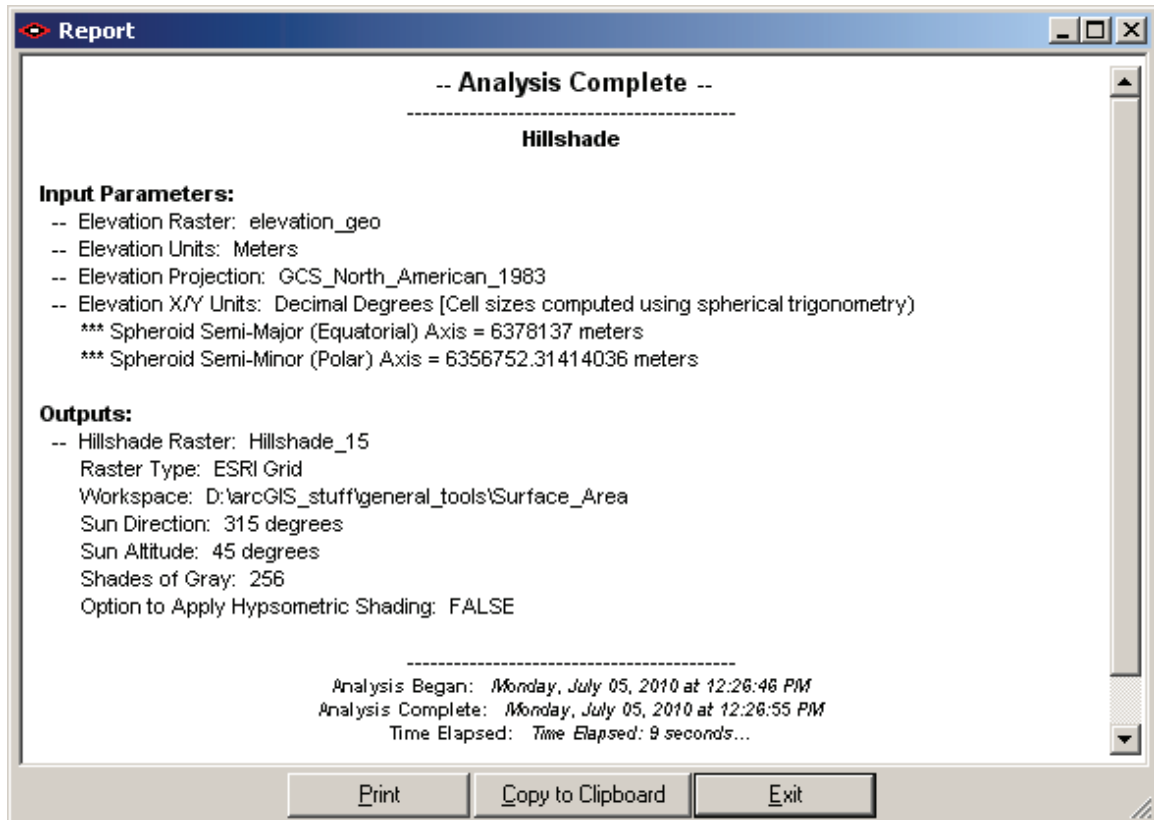
- 3) Select your elevation raster from the listbox of rasters in your map. If you wish to analyze a particular band of a multi-band raster, then click the “All Rasters” tab and select the specific band you wish to analyze.
- 4) Specify the elevation units of your dataset. The tool is currently written to only accept elevation values in meters or feet because those are by far the most common. Please contact the author (jeffj@jennessent.com) if you need additional elevation unit options.
- 5) Specify where to save your hillshade raster.
- 6) Identify the direction and inclination to the illumination source (i.e. the sun).

- 7) Specify how many hillshade levels you wish to generate. If you are interested in a standard gray-scale hillshade, then 256 hillshade levels is typically the maximum useful number of gray levels because most displays and printers do not show more than that. If you want to shade the values over multiple colors, then you have the option to spread the hillshade values over as large a range of values as you wish.
- 8) Choose whether or not to apply hypsometric shading. This option darkens the lower elevation based on the strength value you set. Please refer to the discussion on hypsometric tinting above for more information on this option.
- 9) If desired, enter a vertical exaggeration factor. This option adjusts the vertical relief of the landscape in order to reduce or exaggerate the hillshade. Each elevation value will be multiplied by the exaggeration factor before the hillshade is generated.


An exaggeration value of 1 will use the actual landscape values. A value of 0 will produce a hillshade of a perfectly flat landscape regardless of the actual topography. A value of 2 will double the vertical relief and a value of 3 will triple it.

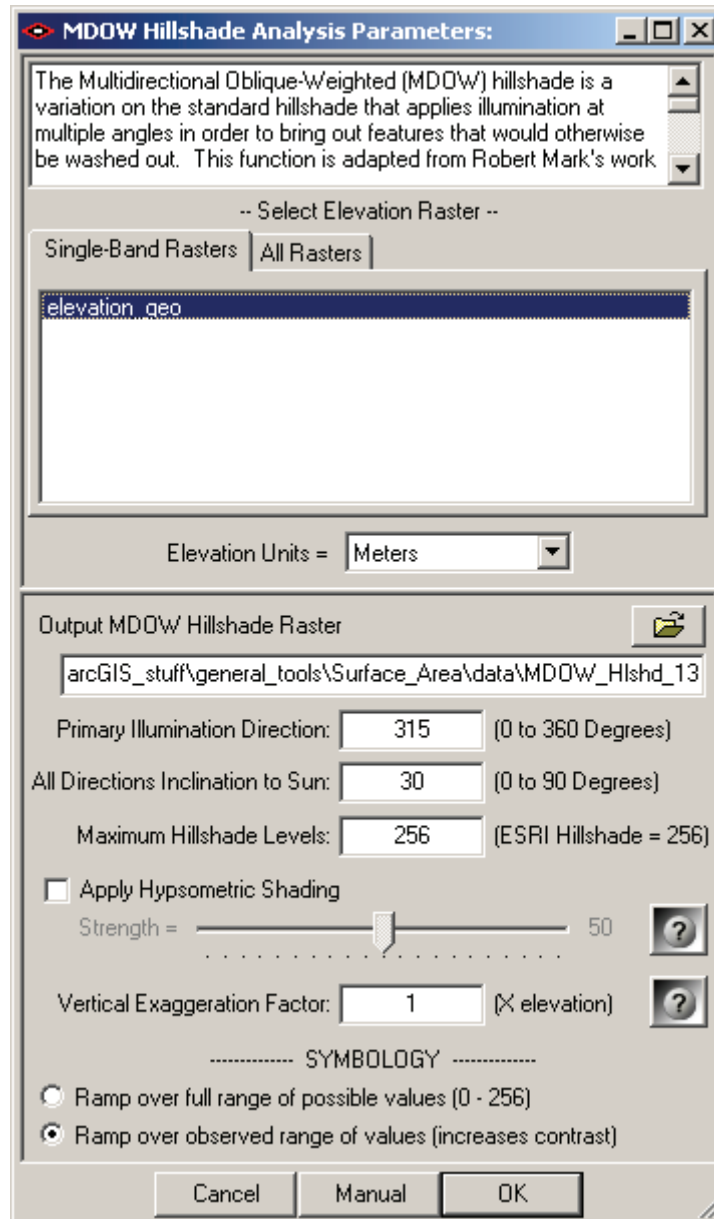
Note: This option is similar, but not identical, to the "Z-Factor" option in the standard ArcGIS hillshade function. Mathematically it is the same, but the ArcGIS "Z-Factor" is primarily intended to allow you to enter a conversion factor for your elevation units, if your elevation units are different than your horizontal units. This Hillshade tool automatically adjusts your elevation units to match your horizontal units, even if your horizontal units are in decimal degrees, so you should only use this option if you want to exaggerate or minimize the hillshade effect. You should **not** enter a conversion factor in this tool.

- 10) **Symbology:** The hillshade will always ramp from black to white, and this option lets you specify whether the minimum and maximum values will be based on your specified range or whether they are taken from the observed range of calculated values. For example, you may specify a maximum of 256 levels, but the actual topography might only have hillshade values that range from 30 to 200. This could happen if no raster cells faced either directly toward or directly away from the light source. In this scenario, the first symbology option (to ramp over the full range of possible values) will produce a raster layer symbology that ramps from 0 to 256 while the second option will only ramp from 30 to 200. The first option tends to look more realistic given the specified lighting source, and the second option tends to have higher contrast.
- 11) Click 'OK' and the tool will generate your hillshade raster for you. Upon completion, it will provide you with a report detailing the analysis parameters:



USING THE MULTIDIRECTIONAL OBLIQUE-WEIGHTED (MDOW) HILLSHADE TOOL:

- 12) Load your elevation raster into ArcGIS.
- 13) Click the *MDOW Hillshade* button  to open the “MDOW Hillshade Analysis Parameters” dialog:



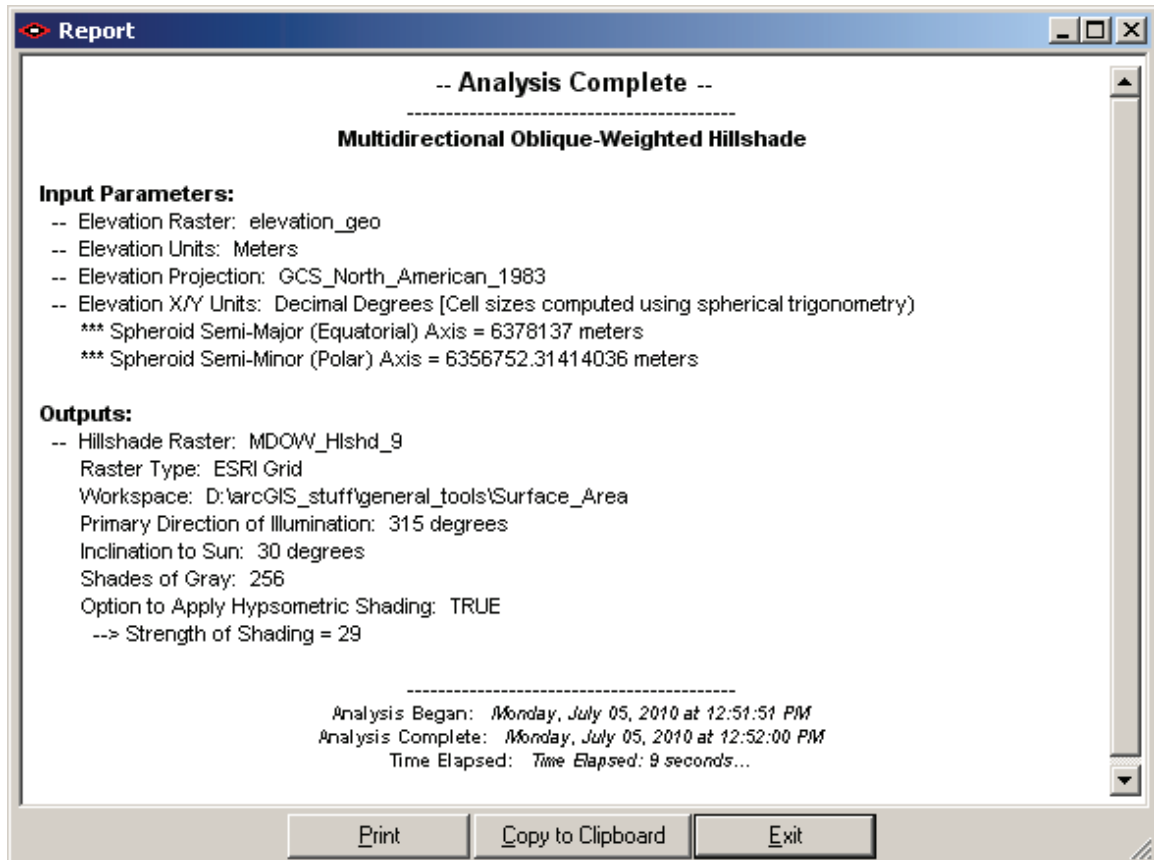
- 14) Select your elevation raster from the listbox of rasters in your map. If you wish to analyze a particular band of a multi-band raster, then click the “All Rasters” tab and select the specific band you wish to analyze.
- 15) Specify the elevation units of your dataset. The tool is currently written to only accept elevation values in meters or feet because those are by far the most common. Please contact the author (jeffj@jennessent.com) if you need additional elevation unit options.
- 16) Specify where to save your MDOW hillshade raster.

- 17) Identify the direction and inclination to the illumination source (i.e. the sun). This tool will generate the 4 MDOW illumination sources around this primary illumination direction.
- 18) Specify how many hillshade levels you wish to generate. If you are interested in a standard gray-scale hillshade, then 256 hillshade levels is typically the maximum useful number of gray levels because most displays and printers do not show more than that. If you want to shade the values over multiple colors, then you have the option to spread the hillshade values over as large a range of values as you wish.
- 19) Choose whether or not to apply hypsometric shading. This option darkens the lower elevation based on the strength value you set. Please refer to the discussion on hypsometric tinting above for more information on this option.
- 20) If desired, enter a vertical exaggeration factor. This option adjusts the vertical relief of the landscape in order to reduce or exaggerate the hillshade. Each elevation value will be multiplied by the exaggeration factor before the hillshade is generated.

An exaggeration value of 1 will use the actual landscape values. A value of 0 will produce a hillshade of a perfectly flat landscape regardless of the actual topography. A value of 2 will double the vertical relief and a value of 3 will triple it.

Note: This option is similar, but not identical, to the "Z-Factor" option in the standard ArcGIS hillshade function. Mathematically it is the same, but the ArcGIS "Z-Factor" is primarily intended to allow you to enter a conversion factor for your elevation units, if your elevation units are different than your horizontal units. This MDOW Hillshade tool automatically adjusts your elevation units to match your horizontal units, even if your horizontal units are in decimal degrees, so you should only use this option if you want to exaggerate or minimize the hillshade effect. You should **not** enter a conversion factor in this tool.

- 21) **Symbology:** The hillshade will always ramp from black to white, and this option lets you specify whether the minimum and maximum values will be based on your specified range or whether they are taken from the observed range of calculated values. For example, you may specify a maximum of 256 levels, but the actual topography might only have hillshade values that range from 30 to 200. This could happen if no raster cells faced either directly toward or directly away from the light source. In this scenario, the first symbology option (to ramp over the full range of possible values) will produce a raster layer symbology that ramps from 0 to 256 while the second option will only ramp from 30 to 200. The first option tends to look more realistic given the specified lighting source, and the second option tends to have higher contrast.
- 22) Click 'OK' and the tool will generate your MDOW hillshade raster for you. Upon completion, it will provide you with a report detailing the analysis parameters:



HILLSHADING METHODS:

Both hillshade options used by this extension use the same algorithm that ArcGIS Spatial Analyst uses to calculate a hillshade. This algorithm is a straightforward and intuitive method that calculates the shading level based on the deviation of the surface normal to the illumination source. If the surface normal (i.e. a line pointing directly perpendicular out from the landscape surface) points directly toward the sun, then the cell receives full illumination. As the surface normal deviates, then the illumination decreases as the cosine of the deviation angle. When the deviation angle reaches 90 degrees, then the cell no longer receives any illumination.

The ArcGIS help files do not give the source of the equation, but it is identical to *Wiechel's Projected Incident Angle* (see section XXXII in Horn [1991]). To maintain consistency with the existing ArcGIS hillshade tool, this tool implements the Wiechel's Projected Incident Angle function and uses Horn's [1991] equations to calculate slope and aspect. In order to calculate slope and aspect correctly for latitude / longitude DEMs, this tool uses the modifications to Horn's method described in the section on *Slope and Aspect Methods*.

The function below returns hillshade values ranging from 1 (at full illumination) to -1 (where cell faces directly away from the sun). In practice, all hillshade values ≤ 0 are set to 0.

$$\text{Hillshade} = \cos i = \cos(\theta)\cos(\theta_0) + \sin(\theta)\sin(\theta_0)\cos(\phi - \phi_0)$$

where :

i = Angle of from surface normal to sun, in radians

θ = Angle of inclination from 90° to sun, in radians

$$= (90 - \text{Angle from Horizon to Sun, in degrees}) \left(\frac{\pi}{180} \right)$$

ϕ = Mathematical direction to sun, in radians

$$= ((450 - \text{Compass direction to sun, in degrees}) \bmod(360)) \left(\frac{\pi}{180} \right)$$

θ_0 = Slope of raster cell in radians, as positive value

$$= \arctan \sqrt{G^2 + H^2}$$

ϕ_0 = Mathematical direction of raster cell Aspect, in radians

$$= \text{atan } 2(H, -G) \left(\frac{180}{\pi} \right)$$

$$G = \frac{\delta z}{\delta x} = \frac{\Delta Z_{(\text{Row } 1)} + 2(\Delta Z_{(\text{Row } 2)}) + \Delta Z_{(\text{Row } 3)}}{\text{Length}_{(\text{Row } 1)} + 2(\text{Length}_{(\text{Row } 2)}) + \text{Length}_{(\text{Row } 3)}}$$

$$H = \frac{\delta z}{\delta y} = \frac{\Delta Z_{(\text{Col } 1)} + 2(\Delta Z_{(\text{Col } 2)}) + \Delta Z_{(\text{Col } 3)}}{\text{Length}_{(\text{Col } 1)} + 2(\text{Length}_{(\text{Col } 2)}) + \text{Length}_{(\text{Col } 3)}}$$

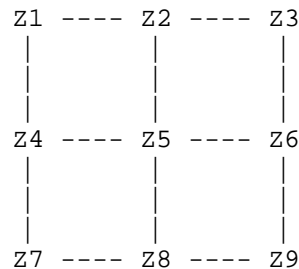
Landscape Curvature

BACKGROUND

Landscape curvature is surprisingly difficult to describe. There are numerous equations and definitions in the literature but both the definitions and equations often conflict with each other. For example, "Plan Curvature" is defined as either the curvature of a cross-section of the landscape intersecting the XY plane, or curvature of the plane intersecting the plane of the slope normal and perpendicular to aspect direction. Some sources add negative signs to the equations, thus inverting the signs of the curvature values, while others do not.

There are two broad approaches commonly used to calculate curvature from raster digital elevation models, which I will name here as the Evans approach and the Zevenbergen & Thorne approach. **Note:** ArcGIS Spatial Analyst uses the Zevenbergen & Thorne approach. This extension uses the Evans approach, for reasons explained below.

Both approaches use a 3-cell x 3-cell moving window and calculate curvature from the 9 raster cells in the window.



The difference between the approaches is that the **Evans** approach fits a curve to the 9 elevation points using a 6-parameter polynomial (Evans 1979), which is only a best fit to the 9 points and does **not** necessarily go through all points. Florinsky (1998a) argues in favor of this approach because it filters out small random errors in the original DEM compilation. Florinsky (1998b) also shows mathematically that the Evans method is more precise than the Zevenbergen & Thorne method. Schmidt et al. (2003) argues that quadratic-based algorithms (like the Evans approach) are more stable than partial quartic algorithms such as Zevenbergen & Thorne's. Schmidt also states that partial quartic algorithms should only be used with highly accurate data.

Zevenbergen & Thorne use a higher-order 9-parameter polynomial that does go through all 9 points (Zevenbergen and Thorne 1987). By fitting the curve to all 9 points, Thorne argues that this method is more appropriate in rugged areas with higher curvature (Thorne 1987). If the DEM data are highly accurate, then Schmidt would argue that Zevenbergen & Thorne's method is appropriate.

THE DIFFERENCE BETWEEN THE EVANS AND ZEVENBERGEN & THORNE POLYNOMIALS

This tool uses the **Evans** approach to calculate landscape curvature only because Igor Florinsky (1998a) was kind enough to publish methods adapting Evans' equation to latitude/longitude data, and I am not enough of a mathematician to adapt Zevenbergen & Thorne's equations myself. The landscape curvature tools presented here are therefore built on Florinsky's adaptations of Evans' equations.

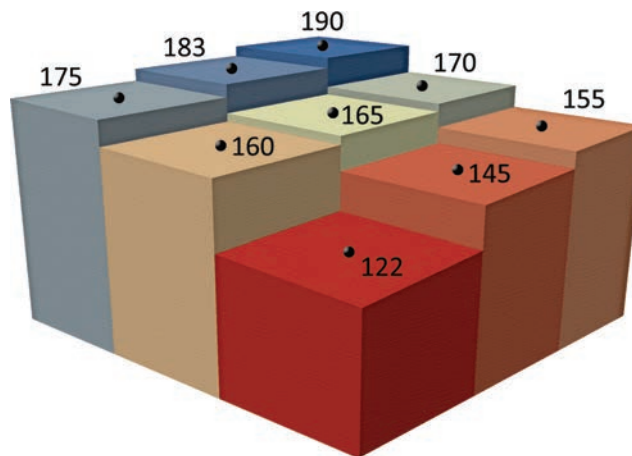
This extension offers options to calculate 7 types of landscape curvature. These options are explained and illustrated in detail in the *Curvature Methods* section below. As a preview, *Plan*, *Tangential*, and *Cross-Sectional* curvature all describe how water would converge or diverge as it flows over a point. *Profile* and *Longitudinal* curvature describe how water would accelerate or decelerate as it flows over a point. *Total* and *General* curvature are general descriptions of how curved the landscape is. *Total* curvature is always a positive value and therefore does not tell you whether the landscape is convex or concave, but it can be used as a measure of landscape

ruggedness. *General* curvature can be positive or negative and therefore does tell you whether the landscape is convex or concave.

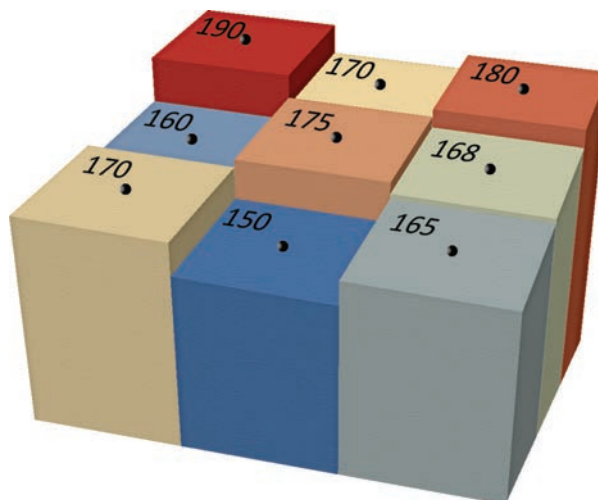
The Evans and Zevenbergen & Thorne polynomials can produce conflicting curvature values. Obviously no method can be guaranteed to be accurate because they are only an estimate of curvature based on 9 sample points in the neighborhood. It is always possible that those 9 points are not good representatives of the actual topography in that area. A review of the literature shows different opinions on which of the two polynomials described here are “better”, and you must judge for yourself which method you prefer. If you prefer the Zevenbergen & Thorne polynomial, then use the standard ArcGIS curvature tools (but remember they are only accurate for projected data). **The tools presented here use the Evans polynomial** and they will work with both projected and unprojected data. **Note:** The actual polynomials are described in detail in “Curvature Methods” below (see p. 75)

To give you a sense of the actual curves produced by these two polynomials, I created two scenarios and generated full 3-dimensional curves using both the Zevenbergen & Thorne and Evans polynomials. The following examples may help you understand the polynomials a little better.

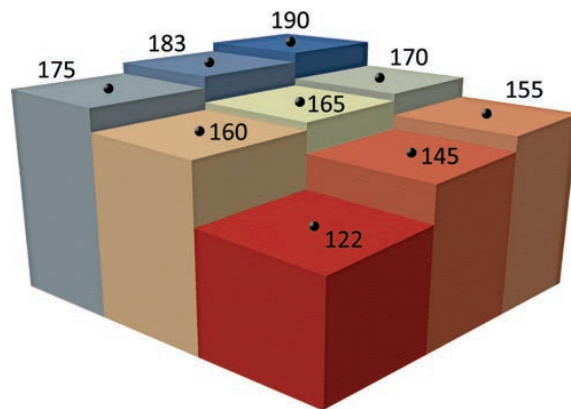
Scenario 1: 30-meter pixel size in a steep area:



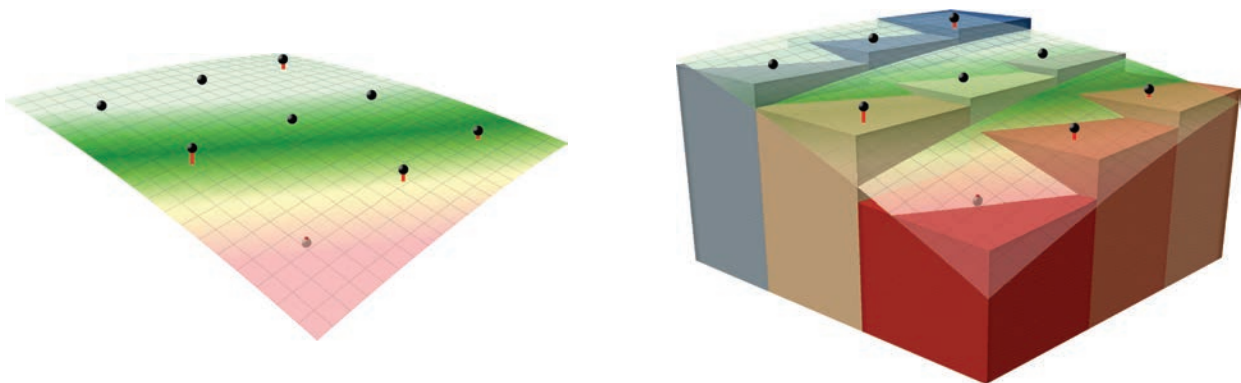
Scenario 2: 30-meter pixel size in complex area, with conflicting curvatures in various directions:



Scenario 1, Evans Polynomial Results



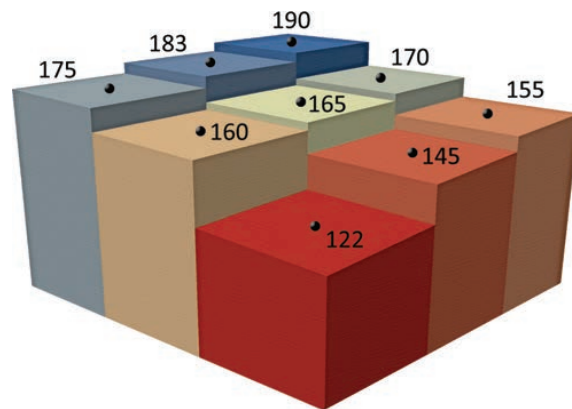
The Evans polynomial produces the following curve over the 9 cells. The red lines indicate how far the curve was from the actual sample points.



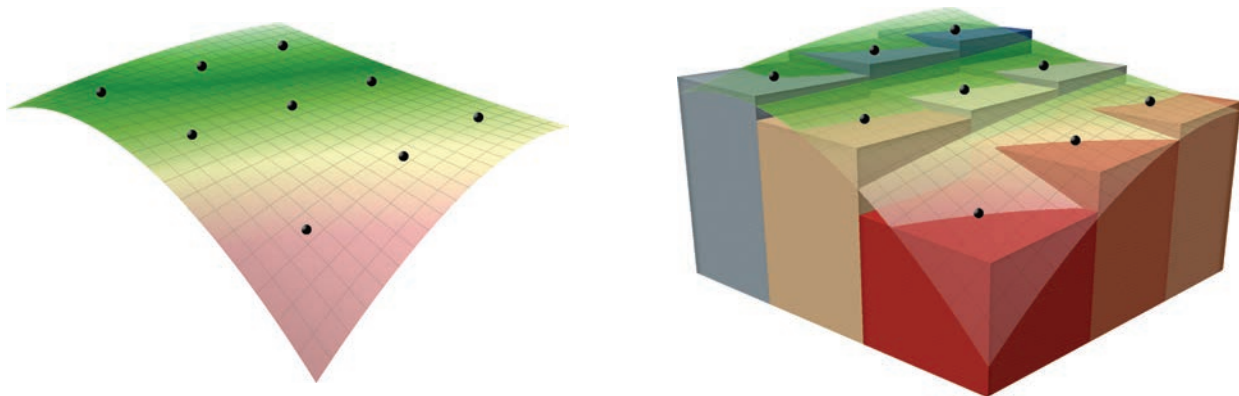
The Evans Polynomial Curvature values for this curve are:

- **Profile Curvature = -0.86**
- **Longitudinal Curvature = -1.78**
 - Negative values suggest that water will **accelerate** as it flows over the central point.
- **Plan Curvature = 0.23**
- **Tangential Curvature = 0.14**
- **Cross-Sectional Curvature = 0.73**
 - Positive values suggest that water will **diverge** as it flows over the central point.
- **Total Curvature = 0.01**
 - General measure of curvature
- **General Curvature = 2.52**
 - Positive value suggests that surface is **convex** at central point.

Scenario 1: Zevenbergen & Thorne Polynomial Results:



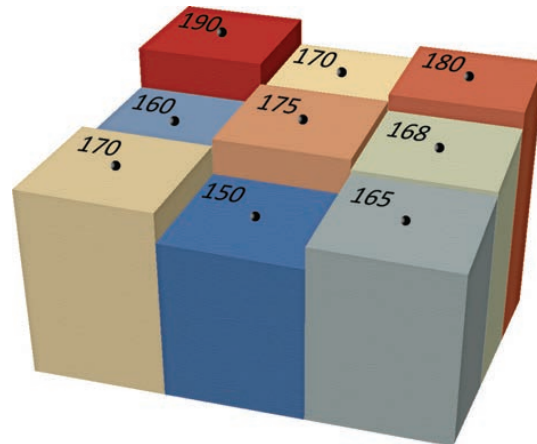
The Zevenbergen & Thorne polynomial produces the following curve over the 9 cells.



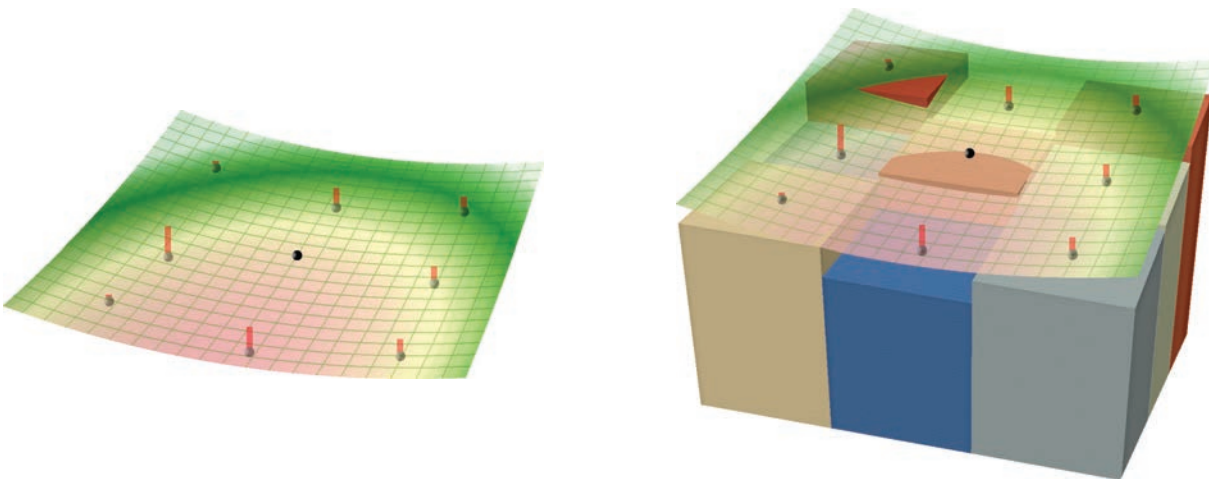
The Zevenbergen & Thorne Polynomial Curvature values for this curve are:

- **Profile Curvature = -0.29**
- **Longitudinal Curvature = -0.45**
 - Negative values suggest that water will **accelerate** as it flows over the central point.
- **Plan Curvature = -0.36**
- **Tangential Curvature = -0.20**
- **Cross-Sectional Curvature = -0.23**
 - Negative values suggest that water will **converge** as it flows over the central point.
- **Total Curvature = 0.01**
 - General measure of curvature
- **General Curvature = 0.22**
 - Positive value suggests that surface is **convex** at central point.

Scenario 2, Evans Polynomial Results



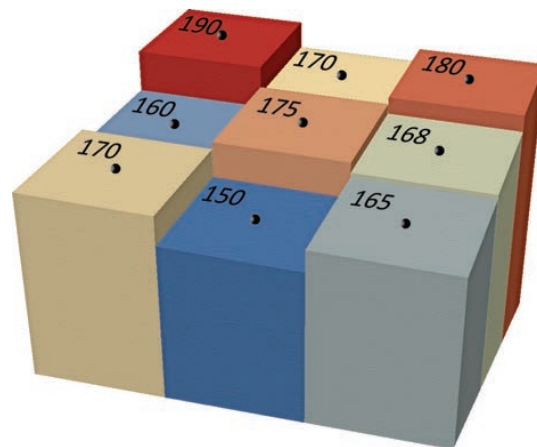
The Evans polynomial produces the following curve over the 9 cells. The red lines indicate how far the curve was from the actual sample points.



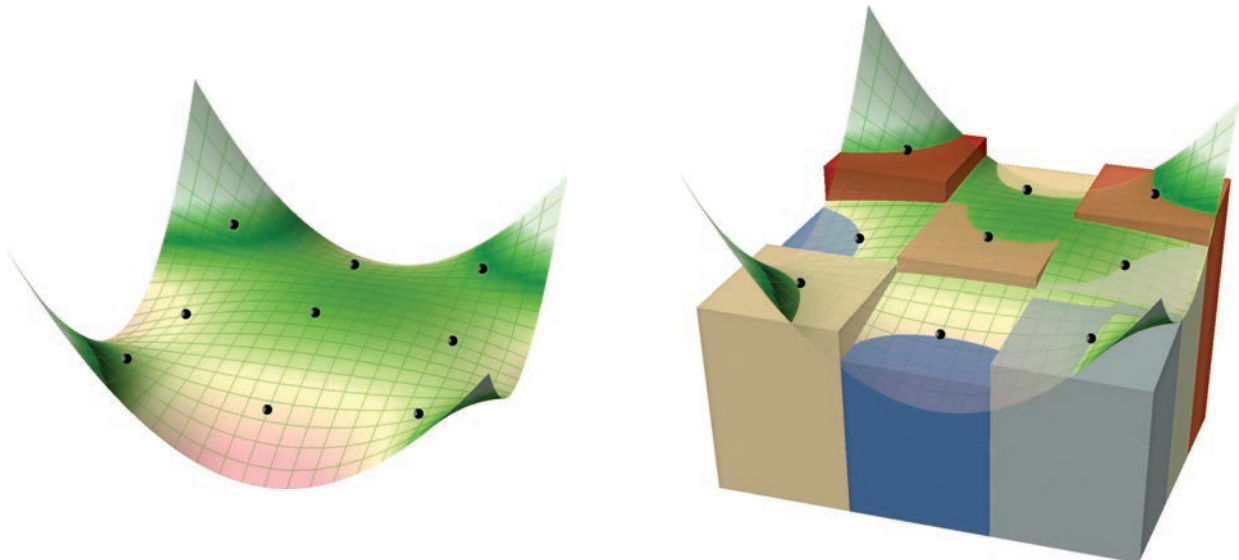
The Evans Polynomial Curvature values for this curve are:

- **Profile Curvature = 0.72**
- **Longitudinal Curvature = 1.47**
 - Positive values suggest that water will **decelerate** as it flows over the central point.
- **Plan Curvature = -5.01**
- **Tangential Curvature = -1.48**
- **Cross-Sectional Curvature = -3.12**
 - Negative values suggest that water will **converge** as it flows over the central point.
- **Total Curvature = 0.03**
 - General measure of curvature
- **General Curvature = -4.59**
 - Negative value suggests that surface is **concave** at central point.

Scenario 2: Zevenbergen & Thorne Polynomial Results:




The Zevenbergen & Thorne polynomial produces the following curve over the 9 cells.

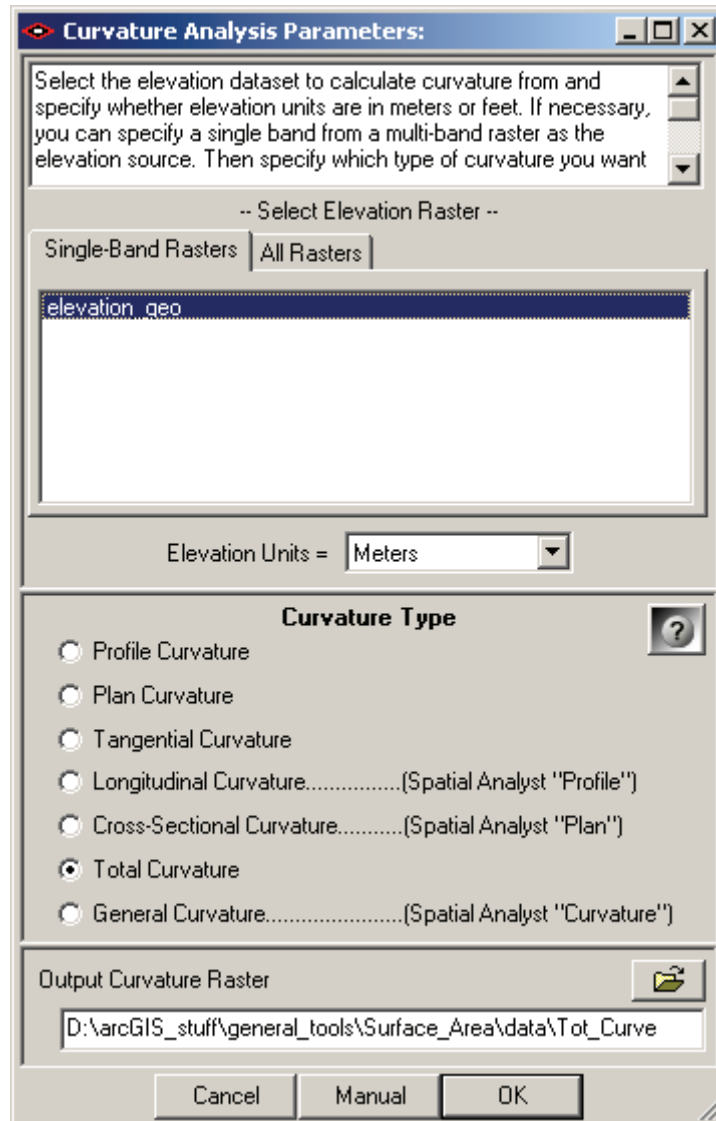


The Zevenbergen & Thorne Polynomial Curvature values for this curve are:

- **Profile Curvature = -1.60**
- **Longitudinal Curvature = -3.31**
 - Negative values suggest that water will **accelerate** as it flows over the central point.
- **Plan Curvature = 3.31**
- **Tangential Curvature = 1.12**
- **Cross-Sectional Curvature = 2.47**
 - Positive values suggest that water will **diverge** as it flows over the central point.
- **Total Curvature = 0.04**
 - General measure of curvature
- **General Curvature = 5.78**
 - Positive value suggests that surface is **convex** at central point.

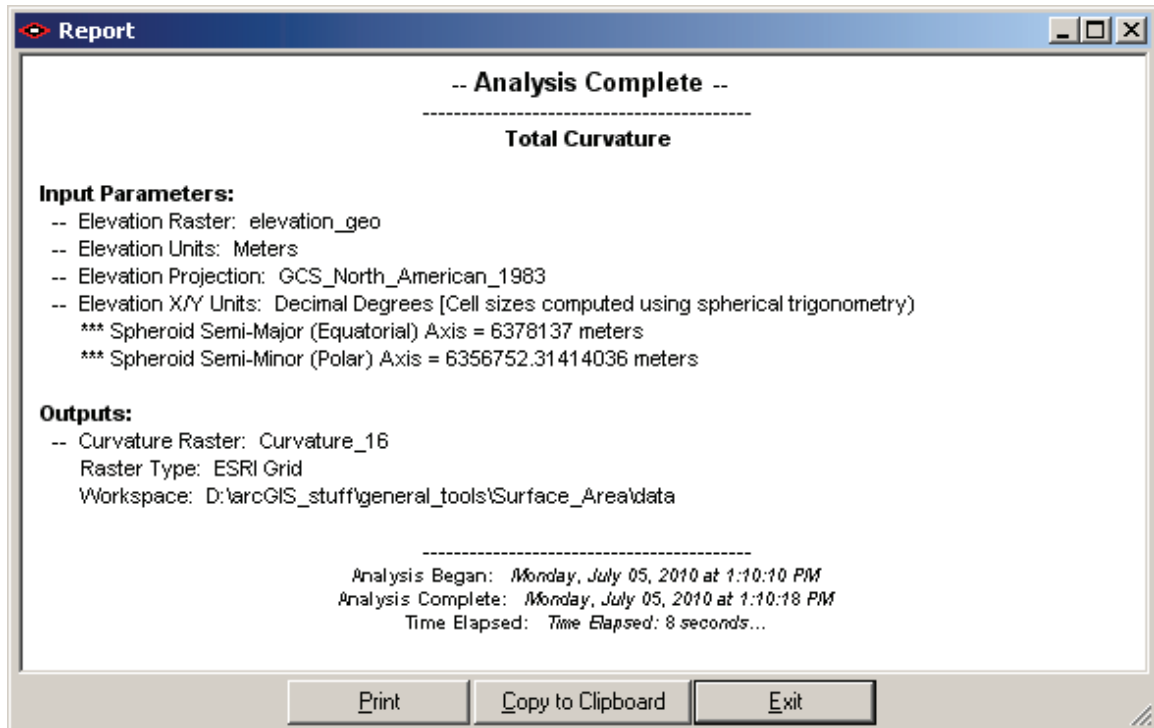
USING THE CURVATURE TOOL:

- 1) Load your elevation raster into ArcGIS.
- 2) Click the *Curvature* button  to open the “Curvature Analysis Parameters” dialog:



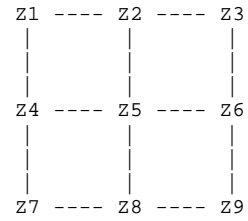
- 3) Select your elevation raster from the listbox of rasters in your map. If you wish to analyze a particular band of a multi-band raster, then click the “All Rasters” tab and select the specific band you wish to analyze.
- 4) Specify the elevation units of your dataset. The tool is currently written to only accept elevation values in meters or feet because those are by far the most common. Please contact the author (jeffj@jennessent.com) if you need additional elevation unit options.
- 5) Specify which type of curvature you wish to create. Please refer to the *Curvature Methods* section below for a detailed discussion of all curvature options. The help button on the dialog also gives illustrations and an explanation of different curvature types.
- 6) Specify where to save your curvature raster.

- 7) Click 'OK' and the tool will generate your curvature raster for you. Upon completion, it will provide you with a report detailing the analysis parameters:



CURVATURE METHODS:

All methods of calculating landscape curvature are based on 1st and 2nd partial derivatives of the curve fitted to the 9 elevation points $Z_1 \dots Z_9$.



The polynomials are:

Zevenbergen & Thorne (1987):

$$Z = Ax^2y^2 + Bx^2y + Cxy^2 + Dx^2 + Ey^2 + Fxy + Gx + Hy + I$$

where :

$$A = \frac{\left[\frac{(Z_1 + Z_3 + Z_7 + Z_9)}{4} - \frac{(Z_2 + Z_4 + Z_6 + Z_8)}{2} + Z_5 \right]}{L^4}$$

$$B = \frac{\left[\frac{(Z_1 + Z_3 - Z_7 - Z_9)}{4} - \frac{(Z_2 - Z_8)}{2} \right]}{L^3}$$

$$C = \frac{\left[\frac{(-Z_1 + Z_3 - Z_7 + Z_9)}{4} + \frac{(Z_4 - Z_6)}{2} \right]}{L^3}$$

$$D = \frac{\left[\frac{(Z_4 + Z_6)}{2} - Z_5 \right]}{L^2}$$

$$E = \frac{\left[\frac{(Z_2 + Z_8)}{2} - Z_5 \right]}{L^2}$$

$$F = \frac{(-Z_1 + Z_3 + Z_7 - Z_9)}{4L^2}$$

$$G = \frac{(-Z_4 + Z_6)}{2L}$$

$$H = \frac{(Z_2 - Z_8)}{2L}$$

$$I = Z_5$$

$L = \text{grid cell size}$

Evans (1979):

$$z = \frac{rx^2}{2} + \frac{ty^2}{2} + sxy + px + qy + u$$

where :

$$r = \frac{\left[Z_1 + Z_3 + Z_4 + Z_6 + Z_7 + Z_9 - 2(Z_2 + Z_5 + Z_8) \right]}{3w^2}$$

$$t = \frac{\left[Z_1 + Z_2 + Z_3 + Z_7 + Z_8 + Z_9 - 2(Z_4 + Z_5 + Z_6) \right]}{3w^2}$$

$$s = \frac{Z_3 + Z_7 - Z_1 - Z_9}{4w^2}$$

$$p = \frac{Z_3 + Z_6 + Z_9 - Z_1 - Z_4 - Z_7}{6w}$$

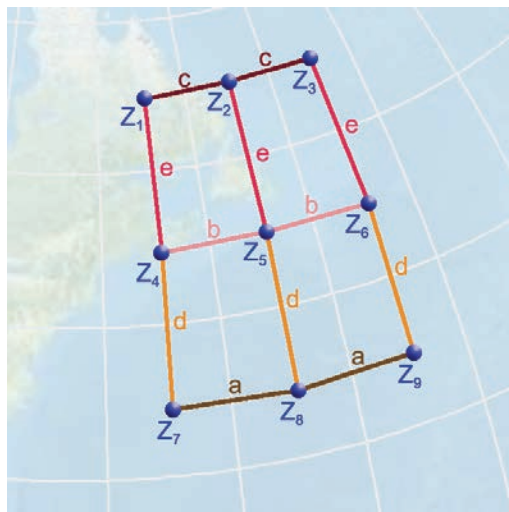
$$q = \frac{Z_1 + Z_2 + Z_3 - Z_7 - Z_8 - Z_9}{6w}$$

$w = \text{grid cell size}$

Note: If you ever wish to test these equations by calculating the elevation value exactly from the modeled curve at any point other than the 9 known points, then be aware the “x” and “y” refer to coordinates relative to the central cell coordinates. For example, if your cell size is 30m, then the x- and y-coordinates for cell Z_7 would be (-30,-30), and the coordinates for cell Z_3 would be (30,30). Z_5 is located at (0,0).

Unfortunately, the parameter definitions for both methods are only valid if the elevation points are equally spaced. Elevation raster data in latitude/longitude coordinates are not evenly spaced. Recall from the discussion of Slope and Aspect that the centerpoints of geographically-projected elevation raster cells actually form trapezoids.

To address this issue, Florinsky (1998a) adapted Evans’ equations for the parameters p , q , r , s and t as follows to account for the true spatial arrangement of elevation locations. Florinsky’s manuscript also shows the derivation for the equations below, plus equations to estimate the accuracy of the parameters. **Note:** If the trapezoid leg lengths a , b , c , d and e are all equal, then Florinsky’s equations return the same values as Evans’ simpler equations above.



Florinsky's Modifications of Evan's Equations (Florinsky 1998a)

$$r = \frac{c^2(Z_1 + Z_3 - 2Z_2) + b^2(Z_4 + Z_6 - 2Z_5) + a^2(Z_7 + Z_9 - 2Z_8)}{a^4 + b^4 + c^4}$$

$$t = \frac{2\left\{\left[d(a^4 + b^4 + b^2c^2) - c^2e(a^2 - b^2)\right](Z_1 + Z_3) - \left[d(a^4 + c^4 + b^2c^2) + e(a^4 + c^4 + a^2b^2)\right](Z_4 + Z_6) + \left[e(b^4 + c^4 + a^2b^2) + a^2d(b^2 - c^2)\right](Z_7 + Z_9) + d\left[b^4(Z_2 - 3Z_5) + c^4(3Z_2 - Z_5) + (a^4 - 2b^2c^2)(Z_2 - Z_5)\right] + 3de(d + e)(a^4 + b^4 + c^4)\right\} + e\left[a^4(3Z_8 - Z_5) + b^4(Z_8 - 3Z_5) + (c^4 - 2a^2b^2)(Z_8 - Z_5)\right] - 2\left[a^2d(b^2 - c^2)Z_8 - c^2e(a^2 - b^2)Z_2\right]}{3de(d + e)(a^4 + b^4 + c^4)}$$

$$s = \frac{c\left[a^2(d + e) + b^2e\right](Z_3 - Z_1) - b(a^2d - c^2e)(Z_4 - Z_6) + a\left[c^2(d + e) + b^2d\right](Z_7 - Z_9)}{2\left[a^2c^2(d + e)^2 + b^2(a^2d^2 + c^2e^2)\right]}$$

$$p = \frac{a^2cd(d + e)(Z_3 - Z_1) + b(a^2d^2 + c^2e^2)(Z_6 - Z_4) + ac^2e(d + e)(Z_9 - Z_7)}{2\left[a^2c^2(d + e)^2 + b^2(a^2d^2 + c^2e^2)\right]}$$

$$q = \frac{\left[d^2(a^4 + b^4 + b^2c^2) + c^2e^2(a^2 - b^2)\right](Z_1 + Z_3) - \left[d^2(a^4 + c^4 + b^2c^2) - e^2(a^4 + c^4 + a^2b^2)\right](Z_4 + Z_6) - \left[e^2(b^4 + c^4 + a^2b^2) - a^2d^2(b^2 - c^2)\right](Z_7 + Z_9) + d^2\left[b^4(Z_2 - 3Z_5) + c^4(3Z_2 - Z_5) + (a^4 - 2b^2c^2)(Z_2 - Z_5)\right] + 3de(d + e)(a^4 + b^4 + c^4)}{3de(d + e)(a^4 + b^4 + c^4)}$$

$$+ e^2\left[a^4(Z_5 - 3Z_8) + b^4(3Z_5 - Z_8) + (c^4 - 2a^2b^2)(Z_5 - Z_8)\right] - 2\left[a^2d^2(b^2 - c^2)Z_8 + c^2e^2(a^2 - b^2)Z_2\right]$$

The partial derivatives necessary for the various Curvature equations are:

$$p = \frac{\delta z}{\delta x} \quad q = \frac{\delta z}{\delta y} \quad r = \frac{\delta^2 z}{\delta x^2} \quad s = \frac{\delta^2 z}{\delta y \delta x} \quad t = \frac{\delta^2 z}{\delta y^2}$$

1st Derivative of Elevation: Slope

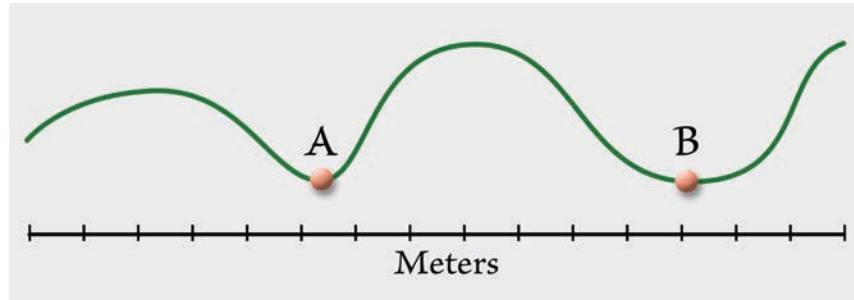
- East-West Gradient = $\frac{\delta z}{\delta x} = G$ (Zevenbergen) = p (Evans)
- North-South Gradient = $\frac{\delta z}{\delta y} = H$ (Zevenbergen) = q (Evans)

2nd Derivative of Elevation: Change of Slope, or Curvature

- X-direction = $\frac{\delta^2 z}{\delta x^2} = D$ (Zevenbergen) = r (Evans)
- Y-direction = $\frac{\delta^2 z}{\delta y^2} = E$ (Zevenbergen) = t (Evans)
- Diagonal = $\frac{\delta^2 z}{\delta x \delta y} = F$ (Zevenbergen) = s (Evans)

CURVATURE UNITS

In general, curvature units are in $\frac{\text{Radians}}{LU}$, where LU = linear unit (usually meters or feet). In other words, assuming that the central cell has some curvature value at that point, then this curve could be extended to form a full circle. This circle has a radius, and higher curvature values imply smaller circles with smaller radii. If you move along this circle for one linear unit (usually one meter or one foot) and measure the number of radians you have moved, then you have the curvature value $\frac{\text{Radians}}{LU}$.



Curve at Point A

Curve at Point B

Curvature at Point

Extend Curve to Circle

Calculate Circle Radius

Curvature Units are in Radians per Linear Unit

Curvature ≈ 1.2

Curvature ≈ 0.4

Because curvature values are often very small, they are sometimes multiplied by 100 and should therefore be interpreted as $\frac{\text{Radians}}{100LU}$. Some authors treat convexity as positive while others treat concavity as positive, and therefore some will present identical equations except that they might or might not have a negative sign in front.

DIFFERENT TYPES OF CURVATURE

As Porres de la Haza & Pardo Pascual (2002) put it, there are essentially an infinite number of ways to define the curvature of a surface. After studying the literature, this author feels like there might be also an infinite number of conflicting ways to define any particular version of curvature, and countless more ways for all the authors to disagree with each other. This extension includes several variations on curvature as defined by numerous manuscripts.

To illustrate the various versions of curvature, I generated a simple artificial DEM that includes a peak, ridge, valley, sink, and multiple regions where running water would converge, diverge, accelerate or decelerate as it flows over the region. This DEM was generated with the following equation:

$$z = a \sin(x + b \sin(y)) + c \sin(x) + d$$

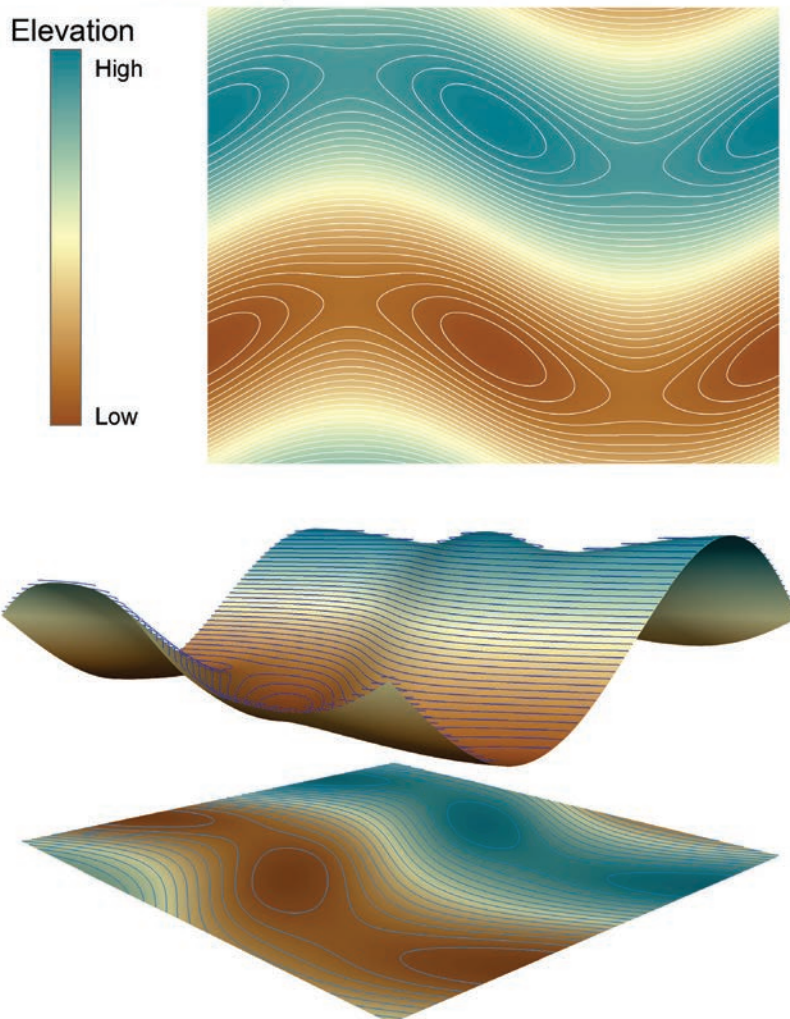
where :

$$x = 0 \dots 2\pi$$

$$y = 0 \dots 2\pi$$

a, b, c, d are constants

Raster Digital Elevation Model

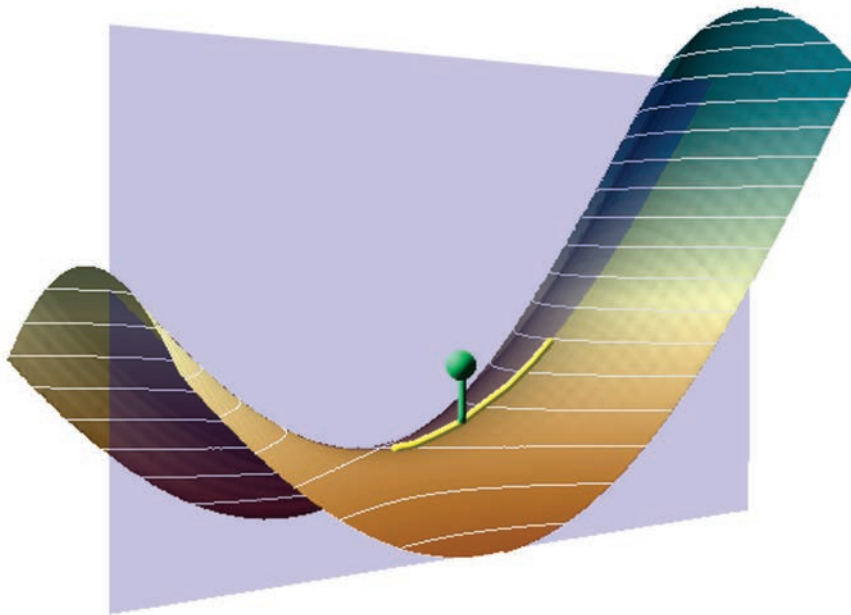


Profile Curvature (as identified by Moore et al. [1993], Wilson & Gallant [2000], Porres de la Haza & Pardo Pascual [2002] and Schmidt et al. [2003]): The curvature along the line of maximum slope, where the surface is intersected with the plane formed by the aspect and the Z-axis. Evans (1980) defines this as the rate of change of gradient. This value tells you whether flowing substances will accelerate or decelerate as they flow over this curve.

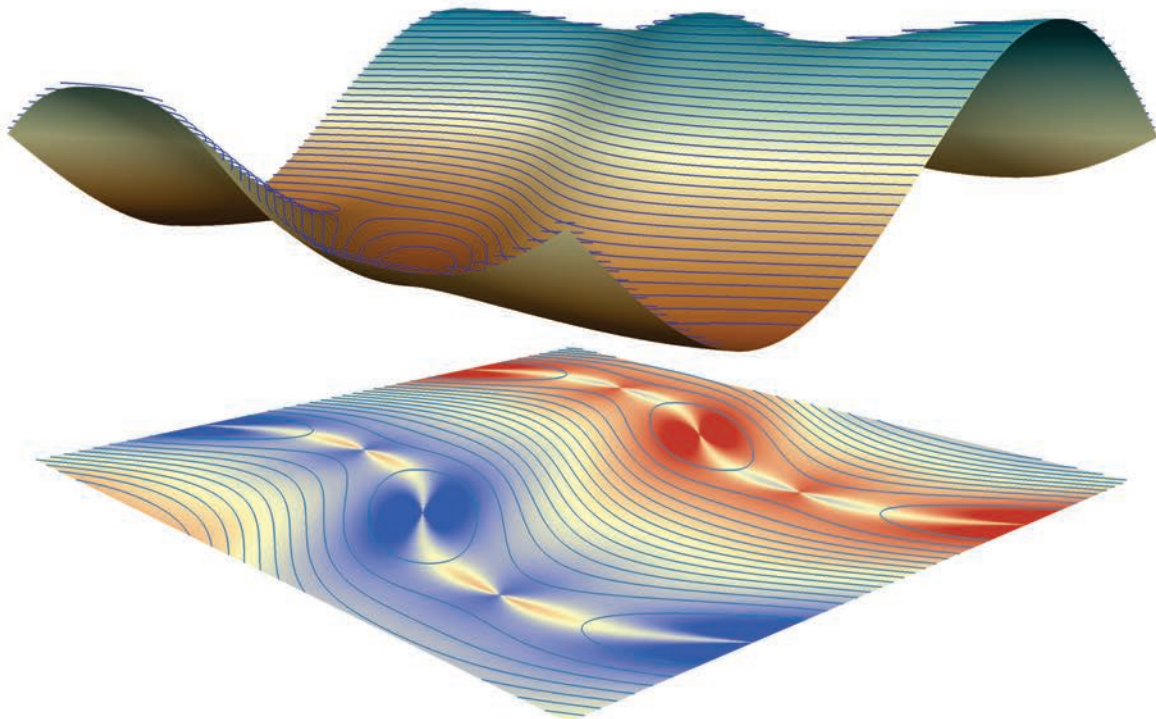
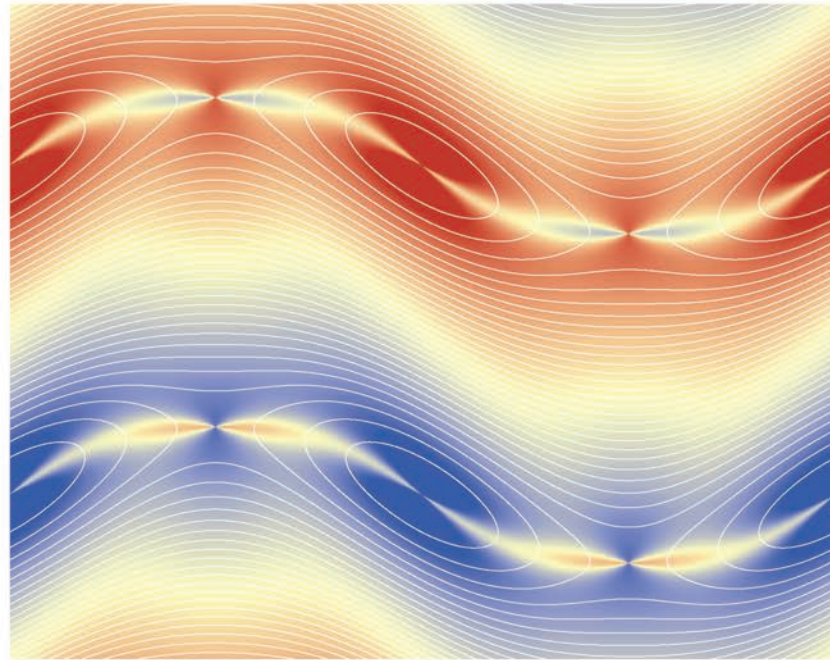
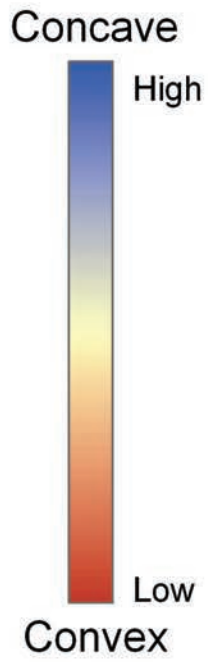
$$\text{Profile Curvature} = -\frac{p^2r + 2pqs + q^2t}{(p^2 + q^2)(\sqrt{1 + p^2 + q^2})^3}, \text{ optionally multiplied by 100}$$

To maintain consistency with ArcGIS curvature functions, profile curvatures are set to positive when the curvature is concave (i.e. when water would decelerate as it flows over this point). Negative values indicate convex curvature where stream flow would accelerate. This value is undefined when the slope is exactly 0 (i.e. when the East-West and North-South gradients p and $q = 0$). Again to maintain consistency with the ArcGIS curvature functions, this tool sets undefined values to 0 and multiplies the final value by 100.

Note: This is referred to as “vertical curvature” by Shary (1995), Florinsky (1998), Florinsky et al. (2002) and Shary et al. (2005). It is also presented as “profile curvature”, but with a slightly different equation by Wood (1996). This is **not** Zevenbergen & Thorne’s definition of profile curvature (and consequently **not** the ArcGIS Spatial Analyst definition). Rather, the Zevenbergen & Thorne / Spatial Analyst “profile” curvature actually uses the equation for *Longitudinal Curvature* (see below).



Profile Curvature



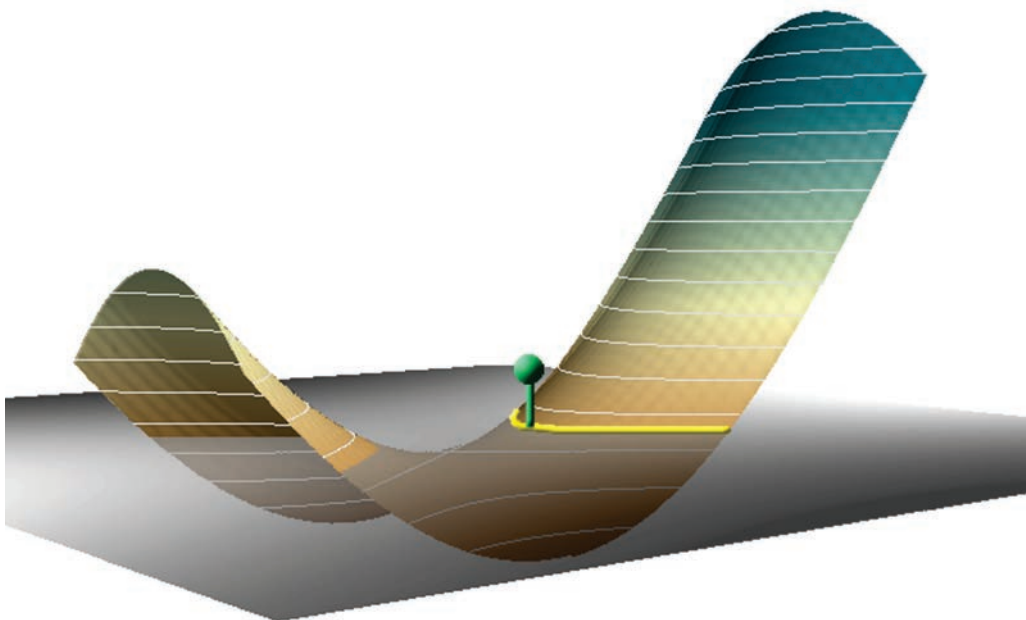
Plan (or Planform) Curvature (as identified by Moore et al. [1993] , Wilson & Gallant [2000] and Porres de la Haza & Pardo Pascual [2002]): The curvature along the line of intersection between the surface and the XY plane. This value tells you whether flowing substances will converge or diverge as they flow over this point. Evans (1980) defines this as “the rate of change of aspect”. This is called “contour” or “horizontal” curvature by Schmidt (2003).

$$\text{Plan Curvature} = \frac{q^2 r - 2 p q s + p^2 t}{\left(\sqrt{p^2 + q^2}\right)^3}, \text{ optionally multiplied by } 100$$

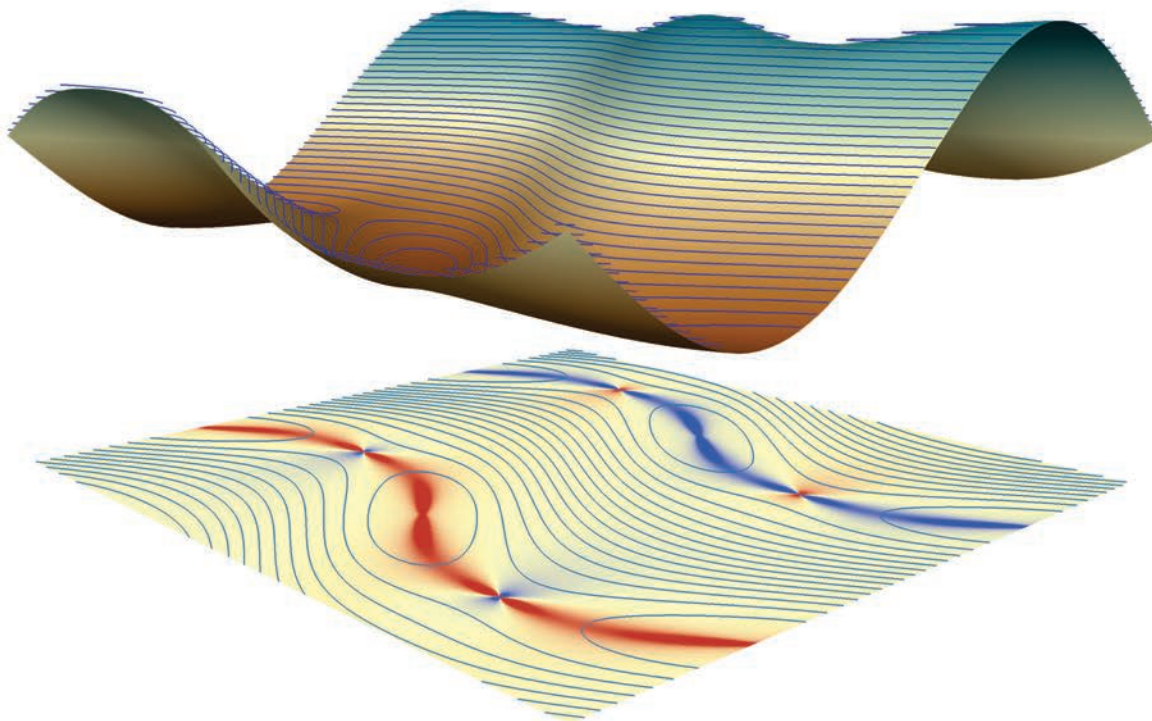
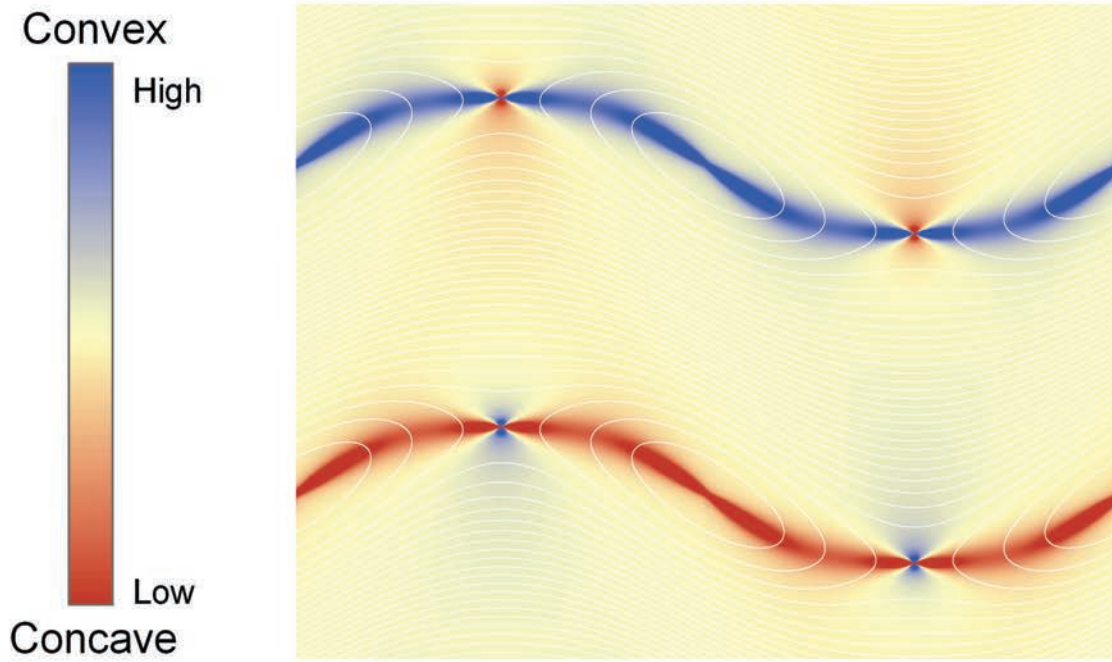
Note: This is **not** Zevenbergen & Thorne’s definition of plan curvature (and consequently **not** the ArcGIS Spatial Analyst definition). Rather, the Zevenbergen & Thorne / Spatial Analyst “plan” curvature uses the equation for *Cross-Sectional Curvature* (see below).

To maintain consistency with ArcGIS curvature functions, plan curvatures are set to positive when the curvature is convex (i.e. when water would diverge as it flows over this point). Negative values indicate concave curvature where stream flow would converge. This value is undefined when the slope is exactly 0 (i.e. when the East-West and North-South gradients p and $q = 0$). Again to maintain consistency with the ArcGIS curvature functions, this tool sets undefined values to 0 and multiplies the final value by 100.

Warning: When the slope is almost, but not quite, perfectly flat, this curvature tends to inflate to extremely large values. If this is a problem in your analysis, you might try the Tangential or Cross-Sectional curvature (see below).



Plan Curvature

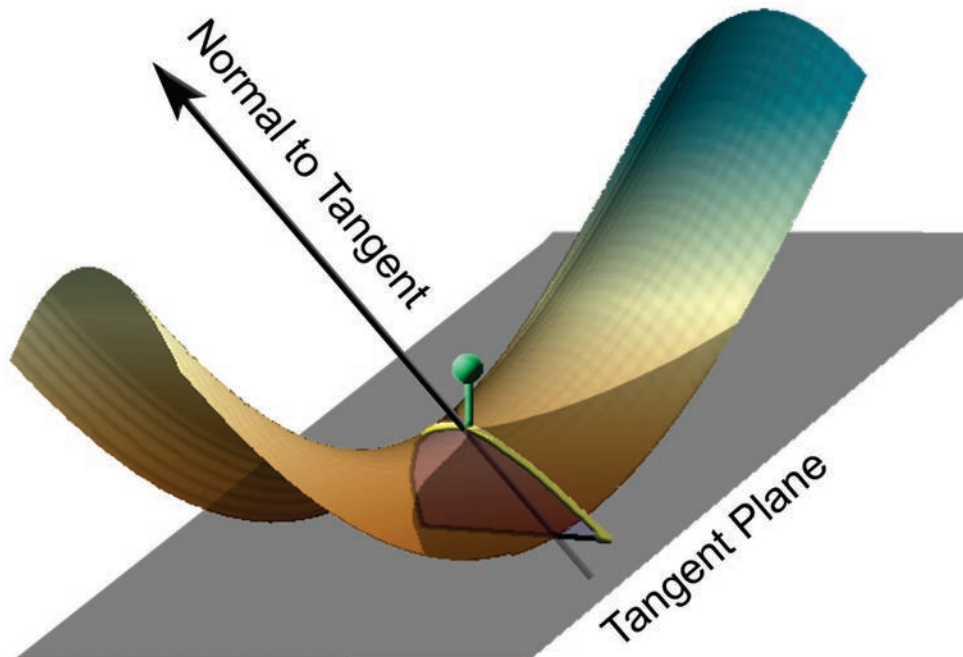


Tangential Curvature (as identified by Wilson & Gallant [2000], Porres de la Haza & Pardo Pascual [2002] and Schmidt et al. [2003]). The curvature along the line orthogonal to the line of steepest gradient. As with Plan curvature, this value tells you whether flowing substances will converge or diverge as they flow over this point.

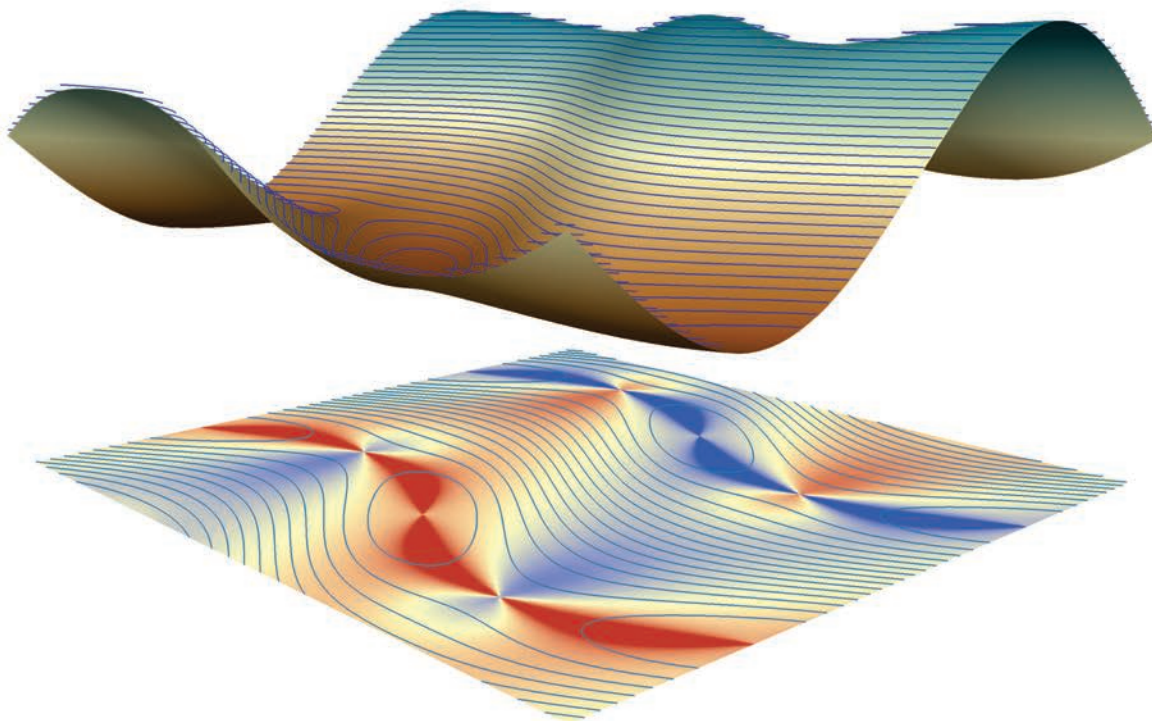
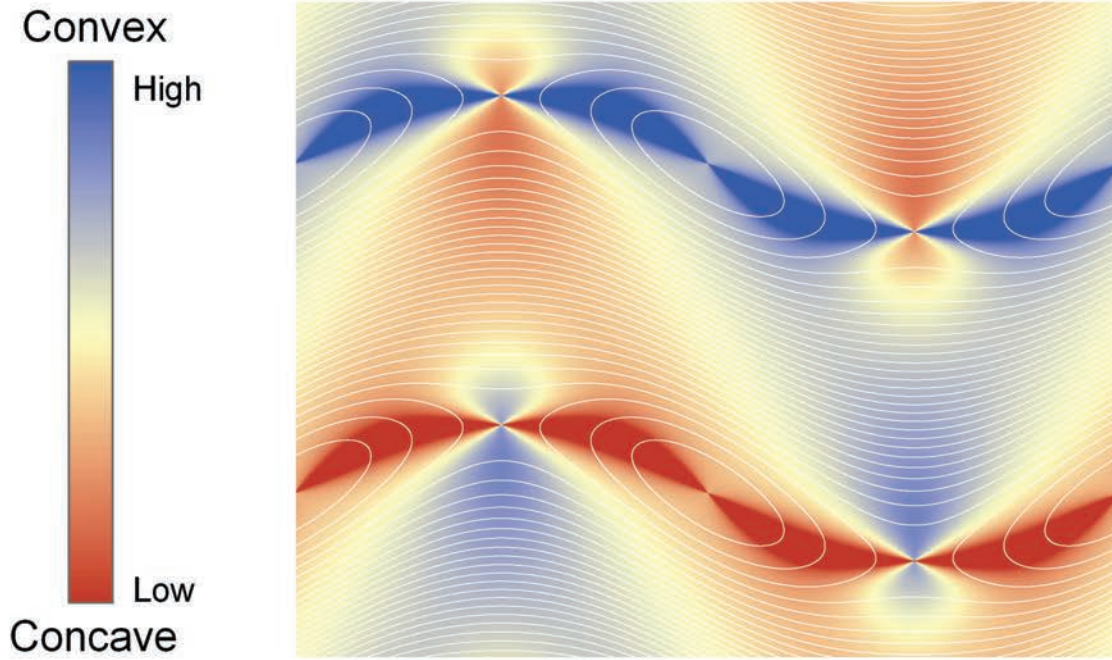
$$\text{Tangential Curvature} = -\frac{q^2 r - 2 p q s + p^2 t}{(p^2 + q^2) \sqrt{1 + p^2 + q^2}}, \text{ optionally multiplied by } 100$$

To maintain consistency with ArcGIS curvature functions, tangential curvatures are set to positive when the curvature is convex (i.e. when water would diverge as it flows over this point). Negative values indicate concave curvature where stream flow would converge. This value is undefined when the slope is exactly 0 (i.e. when the East-West and North-South gradients p and $q = 0$). Again to maintain consistency with the ArcGIS curvature functions, this tool sets undefined values to 0 and multiplies the final value by 100.

Note: This is referred to as “horizontal curvature” by Florinsky (1998), Florinsky et al. (2002) and Shary et al. (2005).



Tangential Curvature



Longitudinal Curvature (as identified by Wood [1996] and Porres de la Haza & Pardo Pascual [2002]): This is conceptually similar to the curvature of the line of intersection between the surface and plane defined by the slope and aspect direction. It is interpreted the same as *profile curvature*, in that it tells you whether a flowing substance will be accelerating or decelerating as it goes over the point.

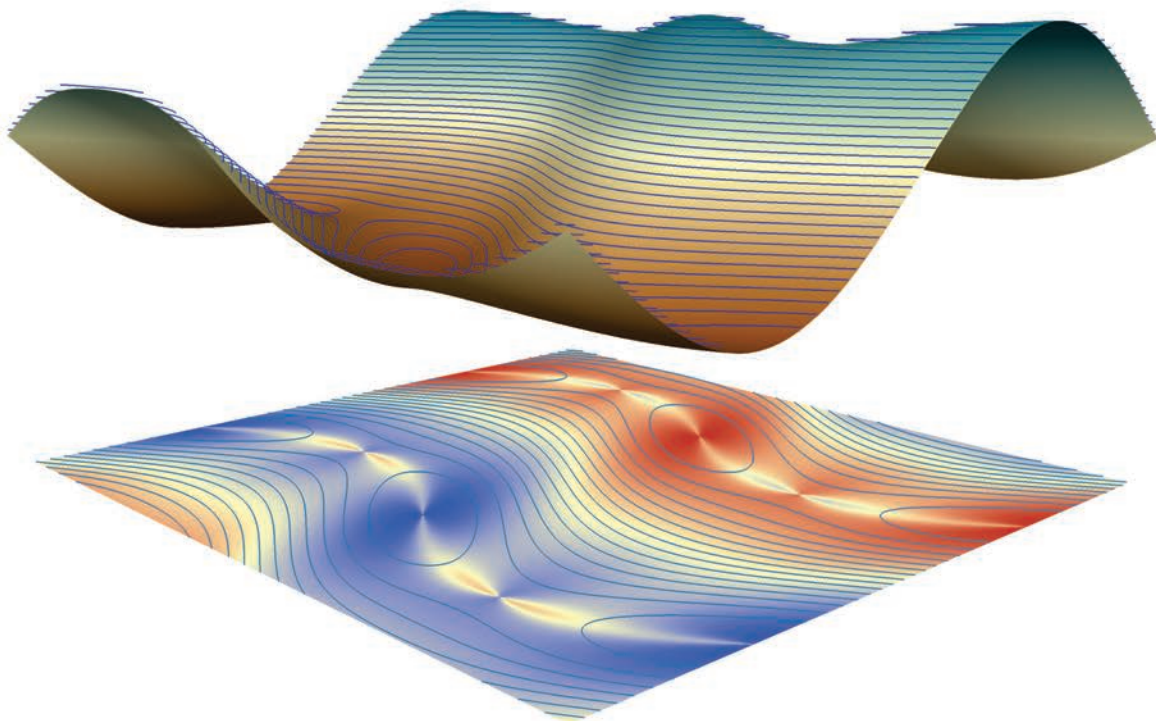
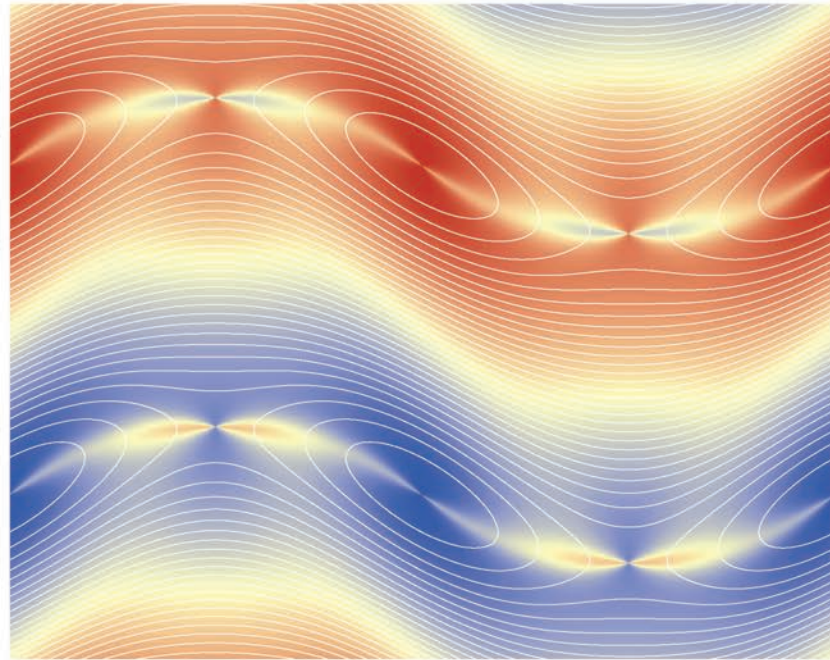
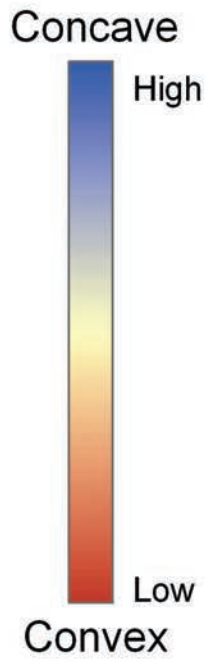
$$\text{Longitudinal Curvature} = -2 \left(\frac{p^2 r + pqs + q^2 t}{p^2 + q^2} \right), \text{ optionally multiplied by 100}$$

To maintain consistency with ArcGIS curvature functions, longitudinal curvatures are set to positive when the curvature is concave (i.e. when water would decelerate as it flows over this point). Negative values indicate convex curvature where stream flow would accelerate. This value is undefined when the slope is exactly 0 (i.e. when the East-West and North-South gradients p and $q = 0$). Again to maintain consistency with the ArcGIS curvature functions, this tool sets undefined values to 0 and multiplies the final value by 100.

Note: This is referred to as “Profile Curvature” by Zevenbergen & Thorne (1987), Thorne et al. (1987) and Moore et al. (1991). This is also the value calculated by the ArcGIS Spatial Analyst Profile Curvature function.

According to Porres de la Haza & Pardo Pascual [2002]) and Zevenbergen & Thorne (1987), this is not a true curvature, but rather a “directional derivative” and does not describe a physical curve on the landscape. Therefore it cannot be graphed like the true *Profile Curvature* above. However, the map of longitudinal curvature values shows that it can be interpreted in a similar manner to profile curvature.

Longitudinal Curvature



Cross-Sectional Curvature (as identified by Wood [1996] and Porres de la Haza & Pardo Pascual [2002]): This is conceptually similar to the curvature of the line of intersection between the surface and plane defined by the slope normal and aspect direction. It is interpreted the same as *plan curvature*, in that it tells you whether a flowing substance will be converging or diverging as it goes over the point.

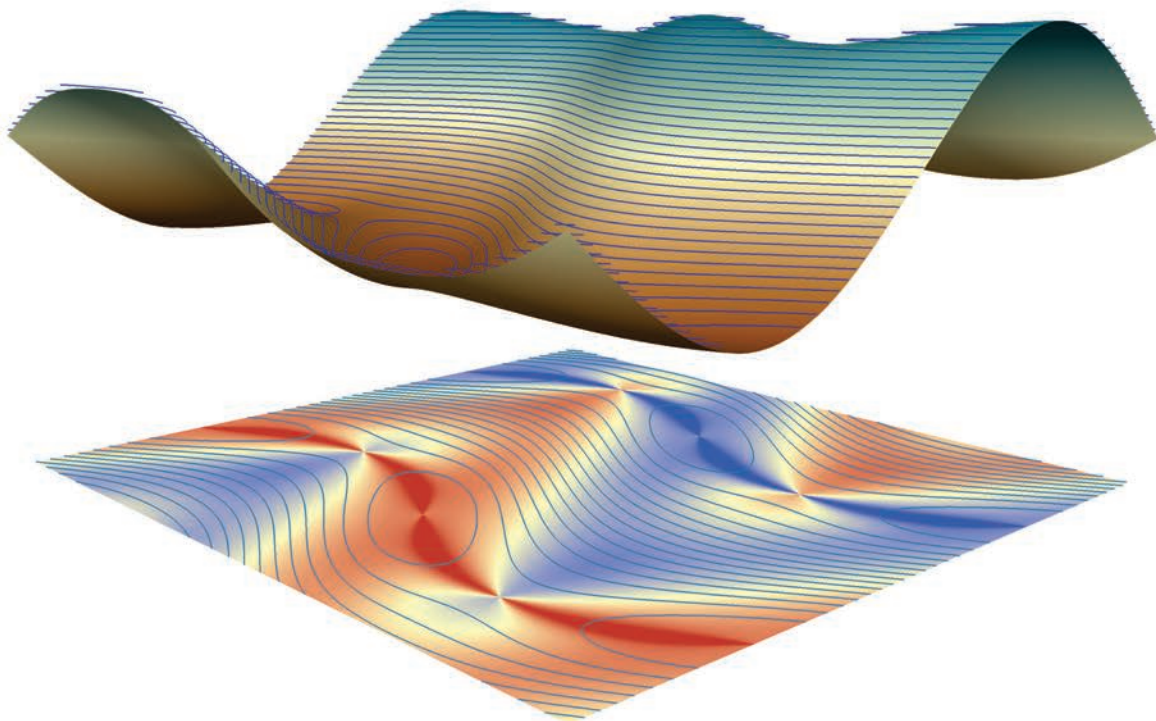
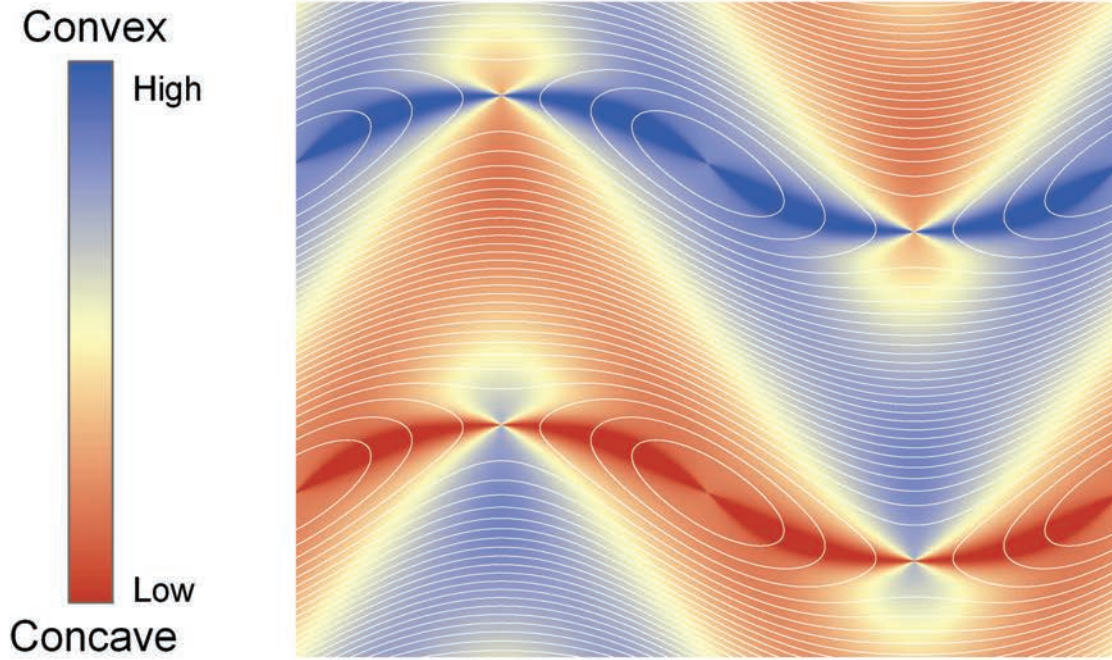
$$\text{Cross-Sectional Curvature} = 2 \left(\frac{q^2 r - pqs + p^2 t}{p^2 + q^2} \right), \text{ optionally multiplied by 100}$$

To maintain consistency with ArcGIS curvature functions, cross-sectional curvatures are set to positive when the curvature is convex (i.e. when water would diverge as it flows over this point). Negative values indicate concave curvature where stream flow would converge. This value is undefined when the slope is exactly 0 (i.e. when the East-West and North-South gradients p and $q = 0$). Again to maintain consistency with the ArcGIS curvature functions, this tool sets undefined values to 0 and multiplies the final value by 100.

Note: This is referred to as “Plan Curvature” by Zevenbergen & Thorne (1987), Thorne et al. (1987) and Moore et al. (1991). This is also the value calculated by the ArcGIS Spatial Analyst Plan Curvature function.

According to Porres de la Haza & Pardo Pascual [2002]) and Zevenbergen & Thorne (1987), this is not a true curvature, but rather a “directional derivative” and does not describe a physical curve on the landscape. Therefore it cannot be graphed like the true *Plan Curvature* above. However, the map of cross-sectional curvature values shows that it can be interpreted in a similar manner to profile or tangential curvature.

Cross-Sectional Curvature



Total Curvature (as identified by Wilson & Gallant [2000] and Schmidt et al. [2003]). This is a general measure of the curvature of the surface itself, rather than the curvature of some line across the surface. The units of this measure are in radians per square meter.

$$\text{Total Curvature} = r^2 + 2s^2 + t^2, \text{ optionally multiplied by 100}$$

Note: This is **not** the value calculated by the ArcGIS *Curvature* function. That value is offered by the *General Curvature* function described below.

This value is always ≥ 0 and can be interpreted as a measure of surface ruggedness. However, note that a value of 0 simply means that the surface is a plane. It does not necessarily mean that the surface is a horizontal plane. To maintain consistency with ArcGIS curvature functions, this function multiplies the final value by 100.

Note: For those who reference Wilson & Gallant (2000), you might notice that they include the following definition of “Total Curvature” on p. 57:

Total curvature is sometimes used as a measure of surface curvature. This curvature measures the curvature of the surface itself, not the curvature of a line across the surface in some direction. It can be positive or negative, with zero curvature indicating that the surface is either flat or the convexity in one direction is balanced by the concavity in another direction, as at a saddle. Total curvature is computed as

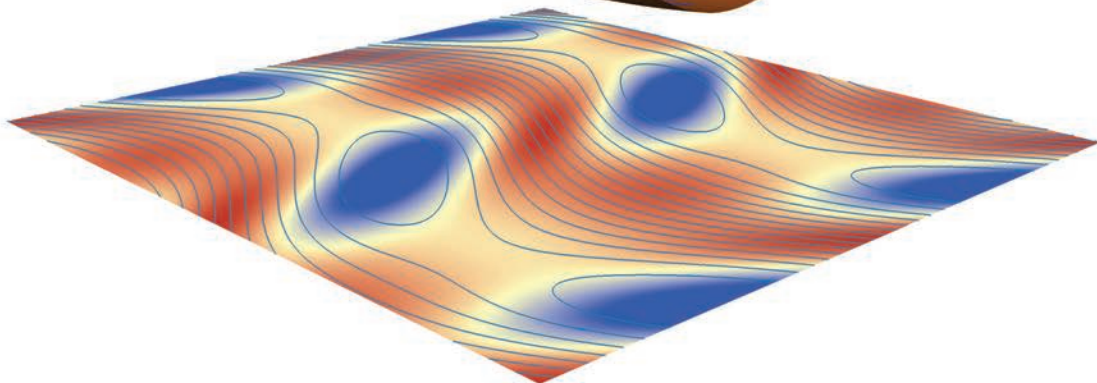
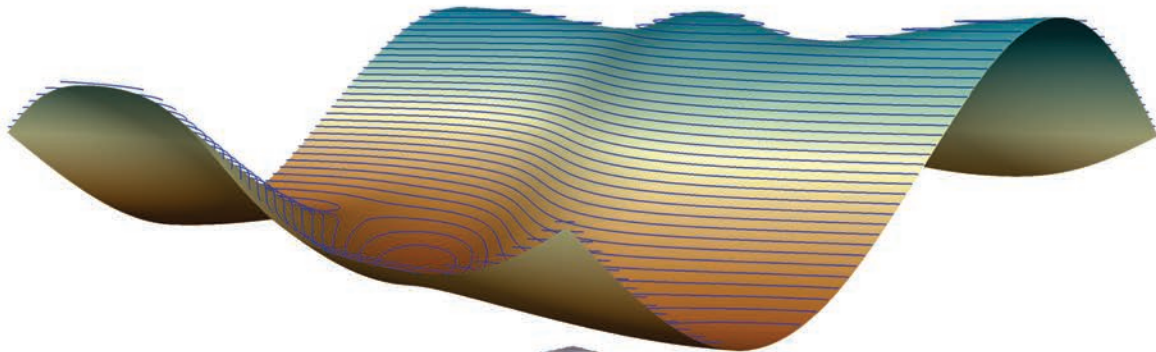
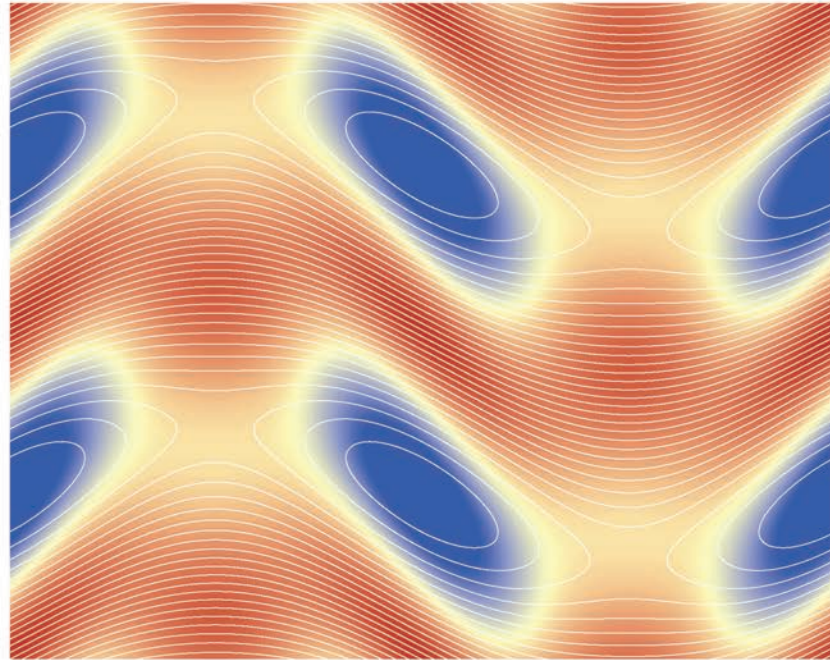
$$K = z^2_{xx} + 2z^2_{xy} + z^2_{yy}$$

If you look close at that equation, there is no way it can ever be a negative value (unless some of the parameters are imaginary values). Any real number squared will be ≥ 0 . Wilson & Gallant’s description much better reflects the ArcGIS definition of curvature (see discussion of “General Curvature” below), while their equation reflects an entirely different type of curvature (see illustration of “Total Curvature” above).

This discrepancy should not be read as a criticism of Wilson & Gallant’s book. It’s a good book and a very useful reference.

Total Curvature

Curvature
High



General Curvature (as identified by Moore et al. 1991, and in a different formulation in Zaslavsky & Sinai 1981): This is a general measure of the convexity of the landscape, where sinks and valleys are considered concave and peaks and high points are considered convex. Convex surfaces have positive values and concave surfaces have negative values. A general curvature value near zero indicates either a flat area or an area where the convexity in one direction is balanced by the concavity in the perpendicular direction, such as on a saddle.

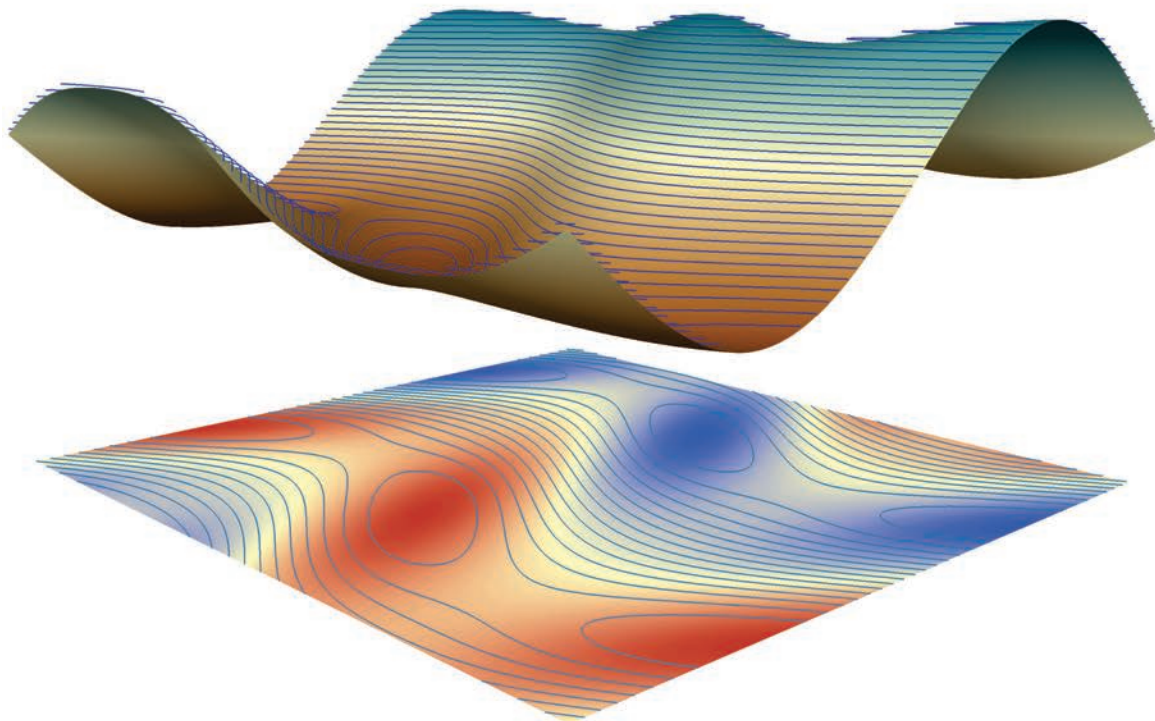
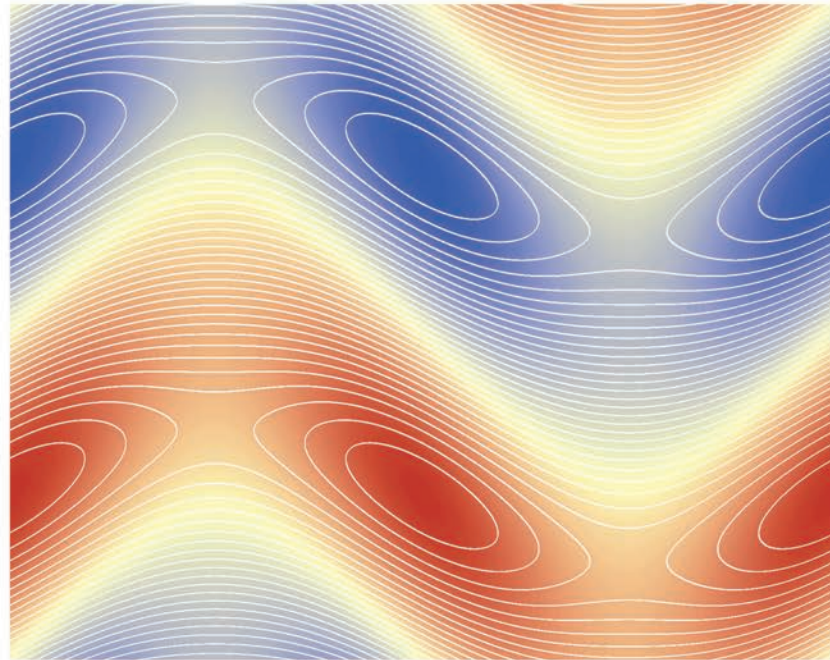
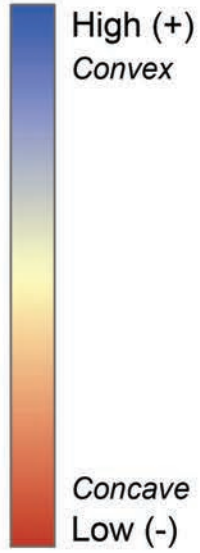
$$\text{General Curvature} = -2(r+t), \text{ optionally multiplied by } 100$$

Note: This is the value calculated by the ArcGIS Spatial Analyst *Curvature* function.

To maintain consistency with ArcGIS curvature functions, general curvature is multiplied by -1. In contrast, Moore et al. (1991) and Zaslavsky & Sinai (1981) define general curvature as equivalent to $2(r+t)$ rather than $-2(r+t)$. This change of sign causes convex surfaces to be positive and concave surfaces to be negative. Also to maintain consistency with the ArcGIS curvature functions, this tool multiplies the final value by 100.

General Curvature

Curvature



Updates

Version 1.0

- Build 1.0.130 (July 7, 2009): Initial Release
- Build 1.0.131 (July 8, 2009): Fixed web site address in "About" dialog.
- Build 1.0.137 (April 8, 2010): Wrote a work-around for an ArcObjects bug in which the rasters would not be correctly written to the hard drive, even when they showed correctly in the map.
- Build 1.0.138 (May 27, 2010): Removed some code that generates a series of message boxes during the analysis.

Version 2.0

- Build 2.0.230 (July 5, 2010):
 - Fixed an error in which Surface Ratio did not calculate correctly unless Surface Area was calculated at the same time.
 - Added functions for Slope, Aspect, Hillshade and Curvature

- Build 2.1.252 (October 10, 2010)
 - Added vertical exaggeration factors to the standard and MDOW hillshade tools.
 - Changed the hillshade symbology to a "Custom Minimum-Maximum Stretch" with options for the minimum and maximum values. **Note:** The actual calculated minimum and maximum hillshade values can easily be different than the specified maximum because there is no guarantee that a hillshade raster will have both regions of perfect brightness and perfect darkness.
 - Added option to calculate "General Curvature", in order to replicate the ArcGIS "Curvature" function.
 - Illustrated difference between Evans and Zevenbergen & Thorne Polynomials in manual.
 - Added illustrations and additional text to explain curvature units.

- Build 2.1.254 (October 20, 2010)
 - Updated the "About" dialog.
 - Added ArcGIS 10 installation functions.

- Build 2.1.272 (November 16, 2010)
 - Wrote a workaround for an ArcGIS 10 bug in which the GX Dialog was unable to recognize a GRID filter, causing an "ActiveX component can't create object" error message. This error would only appear in ArcGIS 10, and would always appear when the user clicked the button to specify a new location to save a raster.

- Build 2.1.292 (February 16, 2011)
 - Fixed a bug in which it occasionally crashed with the message "Failed to Create Raster Dataset"

- Build 2.1.292 (February 13, 2011)
 - Wrote additional ArcGIS 10 installation functions to fix problems in Windows Vista and Windows 7, and on 64-bit versions of Windows.

- Build 2.1.298 (June 27, 2011)
 - Fixed a bug in which spheroidal distances were being calculated incorrectly by a small amount because the spheroid flattening was incorrect. It was being calculated as $(a - b) / b$ instead

of $(a - b) / a$. This would cause very minor errors in all analyses done on geographic rasters.

- **Build 2.1.299 (July 15, 2011)**
 - Updated internal functions to allow extension to remember data between sessions of an ArcGIS MXD document.
- **Build 2.1.305 (February 1, 2012)**
 - Fixed a rare bug that would cause the tool to crash with an Automation Error message when any tool was clicked.
- **Build 2.1.305 (June 11, 2012)**
 - Fixed a bug that would cause the tool to crash if a geographic raster that touched the north or south pole was analyzed.
 - Updated code to clear memory better if Surface Area tool crashed.
 - Fixed a bug in which the tool would crash if a raster had a NaN value for NoData cells.
- **Build 2.1.348 (January 19, 2013)**
 - Fixed a bug that caused incorrect calculations when input raster had a coordinate system that did not use meters. This bug affected the Surface Area, Surface Ratio, Curvature, Hillshade, MDOW Hillshade, Aspect and Slope tools.
 - Added a tool to apply a traditional 9-category directional Aspect symbology.
- **Build 2.1.375 (May 12, 2013)**
 - Fixed a bug in the code that generated default unique filenames for rasters. This would crash if it tried to generate a filename inside a file geodatabase.
 - Fixed a bug in various dialogs that would trigger a crash if you typed in an invalid path for the new filename.

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